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ACIRF User's Guide for the General Model (Version 3.5)

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13. ABSTRACT *(Maximum 200 words)*

Design and evaluation of Radio Frequency (RF) systems that must operate through ionospheric disturbances require an accurate channel model. Such a model can be used to construct realizations of the received signal for use in digital simulations of transionospheric communications links and radars or hardware channel simulators. This report describes the Fortran program ACIRF (Version 3.5) that generates realizations of the channel impulse response functions at the outputs of multiple antennas with arbitrary beamwidths, pointing angles, and relative positions. A general model that varies smoothly between the frozen-in and turbulent models is used for the temporal fluctuations of the impulse response functions. Examples are given which illustrate the effects on the received signal of the variation from the turbulent to frozen-in models, and the effects of antennas with arbitrary sizes and pointing angles. ACIRF inputs and outputs are described.

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PREFACE

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CONVERSION TABLE

Conversion factors for U.S. Customary to metric (SI) units of measurement

MULTIPLY ⟶ BY ⟶ TO GET
TO GET ⟵ BY ⟵ DIVIDE

angstrom	1.000000 x E -10	meters (m)
atmosphere (normal)	1.01325 x E +2	kilo pascal (kPa)
bar	1.000000 x E +2	kilo pascal (kPa)
barn	1.000000 x E -28	meter² (m^2)
British thermal unit (thermochemical)	1.054350 x E +3	joule (J)
calorie (thermochemical)	4.184000	joule (J)
cal (thermochemical) / cm^2	4.184000 x E -2	mega joule/m^2 (MJ/m^2)
curie	3.700000 x E +1	*giga becquerel (GBq)
degree (angle)	1.745329 x E -2	radian (rad)
degree Farenheit	$t_K = (t_F + 459.67)/1.8$	degree kelvin (K)
electron volt	1.60219 x E -19	joule (J)
erg	1.000000 x E -7	joule (J)
erg/second	1.000000 x E -7	watt (W)
foot	3.048000 x E -1	meter (m)
foot-pound-force	1.355818	joule (J)
gallon (U.S. liquid)	3.785412 x E -3	$meter^3$ (m^3)
inch	2.540000 x E -2	meter (m)
jerk	1.000000 x E +9	joule (J)
joule/kilogram (J/kg) (radiation dose absorbed)	1.000000	Gray (Gy)
kilotons	4.183	terajoules
kip (1000 lbf)	4.448222 x E +3	newton (N)
kip/$inch^2$ (ksi)	6.894757 x E +3	kilo pascal (kPa)
ktap	1.000000 x E +2	newton-second/m^2 ($N\text{-}s/m^2$)
micron	1.000000 x E -6	meter (m)
mil	2.540000 x E -5	meter (m)
mile (international)	1.609344 x E +3	meter (m)
ounce	2.834952 x E -2	kilogram (kg)
pound-force (lbs avoirdupois)	4.448222	newton (N)
pound-force inch	1.129848 x E -1	newton-meter (N·m)
pound-force/inch	1.751268 x E +2	newton/meter (N/m)
pound-force/$foot^2$	4.788026 x E -2	kilo pascal (kPa)
pound-force/$inch^2$ (psi)	6.894757	kilo pascal (kPa)
pound-mass (lbm avoirdupois)	4.535924 x E -1	kilogram (kg)
pound-mass-$foot^2$ (moment of inertia)	4.214011 x E -2	kilogram-$meter^2$ ($kg \cdot m^2$)
pound-mass/$foot^3$	1.601846 x E +1	kilogram/$meter^3$ (kg/m^3)
rad (radiation dose absorbed)	1.000000 x E -2	**Gray (Gy)
roentgen	2.579760 x E -4	coulomb/kilogram (C/kg)
shake	1.000000 x E -8	second (s)
slug	1.459390 x E +1	kilogram (kg)
torr (mm Hg, 0° C)	1.333220 x E -1	kilo pascal (kPa)

*The becquerel (Bq) is the SI unit of radioactivity; 1 Bq = 1 event/s.
**The Gray (Gy) is the SI unit of absorbed radiation.

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SECTION 1
INTRODUCTION

Satellite communications systems that use transionospheric propagation links may be subject to severe performance degradation when the ionosphere is highly disturbed by high altitude nuclear explosions [*Arendt and Soicher*, 1964; *King and Fleming*, 1980] or by chemical releases [*Davis et al.*, 1974; *Wolcott et al.*, 1978]. During these events, the increased electron concentrations and the irregular structure of the ionization can lead to intense Rayleigh signal scintillation at the radio frequencies (RF) used for satellite communication links and space radars.

Under severe scintillation conditions, the signal incident at the receiver can vary randomly in amplitude, phase, time-of-arrival, and angle-of-arrival. If all frequency components of the signal vary essentially identically with time, the propagation channel is referred to as nonselective or flat fading. When the scintillations exhibit statistical decorrelation at different frequencies within the signal bandwidth, the channel is referred to as frequency selective. Frequency selective scintillations are therefore encountered when the signal bandwidth exceeds the frequency selective bandwidth of the channel. When the scintillations exhibit statistical decorrelation across the face of an aperture antenna, the channel may also be referred to as spatially selective.

Under conditions where the signal is spatially selective, the antenna beamwidth is smaller than the angle-of-arrival fluctuations, and the effect of the antenna is to attenuate the incident signal that is arriving at off-boresight angles. In the spatial domain, the incident electric field is decorrelated across the face of the antenna. The induced voltages in the antenna then do not add coherently as they would for an incident plane wave, resulting in a loss in the gain of the antenna. Because of this angular filtering or spatial selectivity, the second-order statistics of the signal at the output of the antenna will be different from those of the incident signal.

Design and evaluation of radio frequency systems that must operate through ionospheric disturbances resulting from high altitude nuclear detonations require an accurate channel model. This model must include the effects of scintillation caused by the ionosphere and the effects of high gain antennas that may be used to receive the signals. Such a model can then be used to construct realizations of the received signal for use in digital simulations of transionospheric links or for use in hardware channel simulators.

The propagation channel is conveniently represented in terms of the time-varying channel impulse response function $h(\tau,t)$ or, equivalently, by its Fourier transform, the time-varying channel transfer function $H(\omega,t)$. The former is the channel response at time t to an impulse applied at $t-\tau$. The latter is the channel response to a sinusoidal excitation at radian frequency ω. The received signal $r(t)$ is the convolution of the channel impulse response function and the transmitted modulation $m(t)$:

$$r(t) = \int_0^\infty h(\tau,t)\, m(t-\tau)\, d\tau \ . \tag{1}$$

Two limiting models for the temporal behavior of the channel impulse response function have been used in the past. The first limit is the "frozen-in" model wherein the ionospheric structure or striations that cause scintillation act as a cohesive structure that moves relative to the propagation path. The time variations of the received signal are then a straightforward mapping of the diffracted electric field as it drifts past the receiver. Thus in this frozen-in model there is strong coupling between the spatial and temporal variations of the received signal. The second limiting model is a "turbulent" situation wherein striations move differently at different points along the propagation path, causing time variations of the diffracted electric field to become decorrelated at different points in a plane normal to the line-of-sight. The turbulent model therefore decouples the spatial and temporal variations of the received field. *Wittwer* [1989] proposed a "general" model that falls between these two limiting models.

The purpose of this report is to describe the Fortran channel model ACIRF (Antenna/Channel Impulse Response Function) which generates random realizations of the impulse response function at the outputs of multiple antennas. The theory behind the impulse response function for the general model is discussed in *Dana* [1991]. Channel simulation techniques without antennas have been described by *Wittwer* [1980] and *Knepp and Wittwer* [1984]. The former reference contains a listing of the original CIRF (Channel Impulse Response Function) program developed by Dr. Leon A. Wittwer of the Defense Nuclear Agency. Simulation techniques were extended to include antenna effects in *Dana* [1986, 1989].

The latest version of ACIRF (version 3.5) is based on the general model with antenna aperture effects. A new version of CIRF contains the older frozen-in and turbulent models without antenna effects. RCIRF (Radar/Channel Impulse Response Function) is a code based on ACIRF that produces the impulse response functions for a monopulse radar and two-way propagation through the ionosphere [*Dana*, 19XX]. All three codes use a standard output file format.

Section 2 of this report briefly summarizes the theory behind ACIRF. The Generalized Power Spectral Density (GPSD) function, which describes the second order statistics of the channel impulse response function, is given. Antenna beam profile parameters (beamwidths and pointing angles) are defined, and the effects of the antenna on the received signal statistics are described. Channel simulation techniques used in ACIRF are outlined in Section 3. Examples of the output of a matched filter that illustrate the effects of scintillation and antennas on the received signal are in Section 4. The input and output of the ACIRF and CIRF Fortran codes are discussed in Section 5.

SECTION 2
THEORY

The starting point for channel modeling is the generalized power spectral density (GPSD). This function describes the second-order statistics of the signal incident on the face of an antenna. When scintillation is intense or fully developed, the first-order signal amplitude statistics are well described by the Rayleigh probability distribution, and the signal phase is uniformly distributed over a 2π radian interval. The strong scattering limit can also be obtained by applying the central limit theorem to the superposition of the many randomly scattered waves that comprise the received signal under these conditions. The in-phase and quadrature-phase components of the signal are then independent, zero-mean Gaussian random variables with variances equal to one-half the total signal power. These conditions are both necessary and sufficient for Rayleigh amplitude statistics.

The derivation of the GPSD starts with Maxwell's equations from which the parabolic wave equation is derived (see, for example, *Dana* [1986, 1991]). A necessary condition for the parabolic wave equation to be valid is that the phase perturbation over a distance comparable to a wavelength be small compared to one radian. A sufficient condition is that the angular deviation of the wave relative to the principal propagation path be small compared to one radian. These conditions are generally satisfied whenever attenuation of the propagating wave is not significant.

2.1 MUTUAL COHERENCE FUNCTION.

The parabolic wave equation can be solved to give the received electric field for a specific distribution of the index of refraction. The difficulty is that the index of refraction is a random process. The parabolic wave equation is therefore used to derive an equation for the two-position, two-frequency, two-time mutual coherence function, Γ, of the complex envelope $\boldsymbol{E}$ of the received electric field:

$$\Gamma(x,y,\omega,t) = \langle \boldsymbol{E}(x_1,y_1,\omega_1,t_1)\boldsymbol{E}^*(x_2,y_2,\omega_2,t_2)\rangle \tag{2}$$

where $x = x_1 - x_2$ and $y = y_1 - y_2$ are relative positions in a plane normal to the line-of-sight, $\omega = \omega_1 - \omega_2$ is radian frequency difference, and $t = t_1 - t_2$ is time difference.

The solution of the differential equation for the mutual coherence function describes the second-order statistics of the received signal. The "frozen-in" and "turbulent" models have been used in the past to give the temporal statistics of Γ. In the frozen-in model, the striations in the ionosphere act as a cohesive structure that moves relative to the propagation path. Time variations of the received signal are then a straightforward mapping of the diffracted electric field as it drifts past the receiver antenna. In the turbulent model the striations move differently at different points along the propagation path, causing time variations of the diffracted electric field to become decorrelated at different points about the line-of-sight. The "general" model that falls between these two limiting models. This model has been incorporated into version 3.5 of ACIRF.

The mutual coherence function for the general model is

$$\Gamma(x,y,\omega,t) = \frac{\exp\left[-\frac{1}{2}\left(\frac{\omega}{\alpha\omega_{coh}}\right)^2\right]\exp\left[-(1-C_{xt}^2-C_{yt}^2)\left(\frac{t}{\tau_0}\right)^2\right]}{\left[1+i\,\frac{\omega\Lambda_x}{\omega_{coh}}\right]^{\frac{1}{2}}\left[1+i\,\frac{\omega\Lambda_y}{\omega_{coh}}\right]^{\frac{1}{2}}}$$

$$\times \exp\left[-\frac{\left(\frac{x}{\ell_x}-C_{xt}\frac{t}{\tau_0}\right)^2}{1+i\,\frac{\omega\Lambda_y}{\omega_{coh}}}\right]\exp\left[-\frac{\left(\frac{y}{\ell_y}-C_{yt}\frac{t}{\tau_0}\right)^2}{1+i\,\frac{\omega\Lambda_x}{\omega_{coh}}}\right] \tag{3}$$

where ω_{coh} is coherence bandwidth, C_{xt} and C_{yt} are space-time correlation coefficients ($C_{xt}^2 + C_{yt}^2 = 1$ for the frozen-in model; $C_{xt} = C_{yt} = 0$ for the turbulent model), τ_0 is decorrelation time, ℓ_x and ℓ_y are decorrelation distances, and i is $\sqrt{-1}$. The coherence bandwidth is defined using the GPSD which will be discussed in the next subsection. The delay parameter α is equal to the ratio of the time delay spread caused by diffraction to that caused by refraction. In strong scattering where diffractive effects dominate, the value of α is fairly large. The asymmetry factors Λ_x and Λ_y are given by:

$$\Lambda_x = \left[\frac{2\,\ell_x^4}{\ell_x^4+\ell_y^4}\right]^{\frac{1}{2}} \tag{4a}$$

$$\Lambda_y = \left[\frac{2\,\ell_y^4}{\ell_x^4+\ell_y^4}\right]^{\frac{1}{2}} . \tag{4b}$$

By examining the form of the mutual coherence function, it can be seen that the decorrelation time τ_0 is the $1/e$ point of $\Gamma(0,0,0,t)$, the decorrelation distance ℓ_x is the $1/e$ point of $\Gamma(x,0,0,0)$, and the decorrelation distance ℓ_y is the $1/e$ point of $\Gamma(0,y,0,0)$.

The propagation x–y–z coordinate system, shown in Figure 1, is defined by the line-of-sight direction unit vector $\hat{z}$ and the geomagnetic field $\boldsymbol{B}$ at the altitude in the ionosphere where scattering occurs. The x- and y-axis, defined as

$$\hat{\boldsymbol{x}} = \frac{\boldsymbol{B}\times\hat{z}}{|\boldsymbol{B}|\sin\Phi} \tag{5a}$$

$$\hat{\boldsymbol{y}} = \hat{z}\times\hat{\boldsymbol{x}} \tag{5b}$$

are chosen so that $\ell_x \leq \ell_y$. The penetration angle Φ is the angle between the line-of-sight direction and the geomagnetic field in the ionosphere.

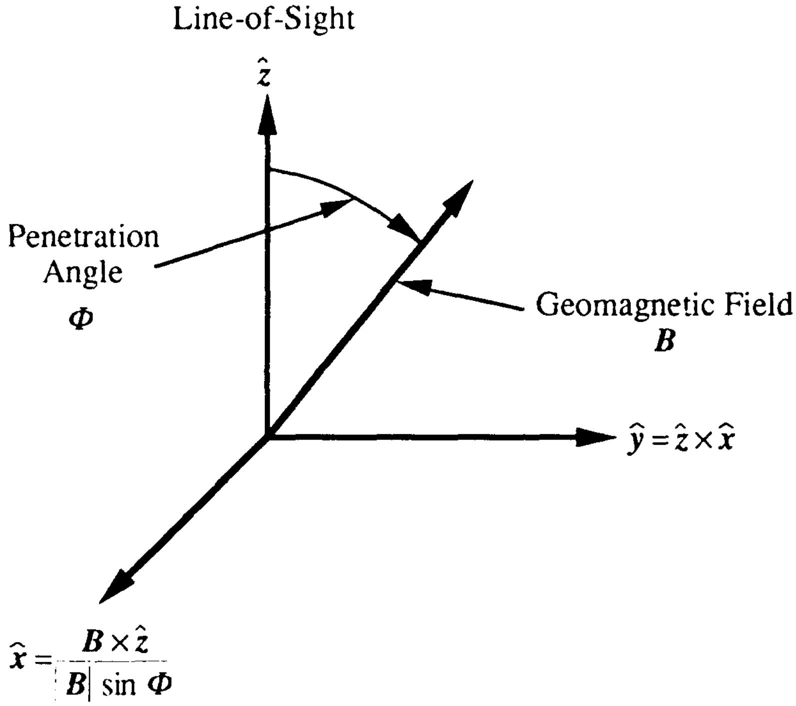

Figure 1. Propagation coordinate system.

In general, both decorrelation distances should be part of the specification of the channel parameters. However, if only the minimum decorrelation distance ℓ_x (also referred to as ℓ_0 in the literature) is specified, the ratio of the decorrelation distances, ℓ_y / ℓ_x, can be estimated from the formula:

$$\frac{\ell_y}{\ell_x} = \sqrt{\cos^2\Phi + q^2 \sin^2\Phi} \ . \tag{6}$$

In this formula q is the axial ratio that is equal to the ratio of the characteristic length of the striations along the field lines to the characteristic thickness of the striations in a direction normal to the field lines. The standard value for q has been 15. However, recent research on the physics of striations indicates that q may be as large as 70 [*Wittwer*, 1988].

2.2 GENERALIZED POWER SPECTRAL DENSITY.

The Fourier transform of the mutual coherence function is the generalized power spectral density of the received signal:

$$S(\boldsymbol{K}_\perp,\tau,\omega_D) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} dt\, \Gamma(x,y,\omega,t) \exp\left[-i(K_x x + K_y y - \omega\tau + \omega_D t)\right] \tag{7}$$

where $\boldsymbol{K}_\perp = (K_x, K_y)$, ω_D is the Doppler radian frequency, and τ is time-of-arrival or delay. The x and y components of the wave vector $\boldsymbol{K}_\perp$ are related to the scattering angles θ_x and θ_y about the x and y axes and the RF wavelength λ as follows:

$$K_x = \frac{2\pi \sin\theta_x}{\lambda} \tag{8a}$$

$$K_y = \frac{2\pi \sin\theta_y}{\lambda} \ . \tag{8b}$$

The quantity

$$S(\boldsymbol{K}_\perp,\tau,\omega_D)\frac{d^2\boldsymbol{K}_\perp}{(2\pi)^2}\,d\tau\,\frac{d\omega_D}{2\pi}$$

is the mean signal power incident on a plane normal to the line-of-sight arriving with $\boldsymbol{K}_\perp$ vector in the interval $\boldsymbol{K}_\perp$ to $\boldsymbol{K}_\perp + d^2\boldsymbol{K}_\perp$; with delay in the interval τ to $\tau + d\tau$, and with Doppler radian frequency in the interval ω_D to $\omega_D + d\omega_D$.

In general, the GPSD can be written as the product of a Doppler term times an angle-delay term:

$$S(\boldsymbol{K}_\perp,\tau,\omega_D) = S_D(\omega_D) S_{K\tau}(\boldsymbol{K}_\perp,\tau) \ , \tag{9}$$

where the Doppler spectrum is

$$S_D(\omega_D) = \frac{\sqrt{\pi}\,\tau_0}{\sqrt{1 - C_{xt}^2 - C_{yt}^2}} \exp\left[-\frac{(\tau_0\omega_D - C_{xt}K_x\ell_x - C_{yt}K_y\ell_y)^2}{4(1 - C_{xt}^2 - C_{yt}^2)}\right] \tag{10}$$

and the angle-delay part of the the GPSD is

$$S_{K\tau}(\boldsymbol{K}_\perp,\tau) = \left[\frac{\pi}{2}\right]^{\frac{1}{2}} \ell_x \ell_y\, \alpha\omega_{coh} \exp\left[-\frac{K_x^2\ell_x^2}{4} - \frac{K_y^2\ell_y^2}{4}\right]$$

$$\times \exp\left\{-\frac{\alpha^2}{2}\left[\omega_{coh}\tau - \frac{\Lambda_y(K_x^2 + K_y^2)\ell_x^2}{4}\right]^2\right\} \ . \tag{11}$$

It is interesting to examine the limits of the Doppler spectrum for the general model to show that this model does indeed encompass both the frozen-in and turbulent models. These limits are:

$$\lim_{\substack{C_{xt} \to 1 \\ C_{yt} \to 0}} S_D(\omega_D) = 2\pi \, \tau_0 \, \delta(\tau_0 \omega_D - K_x \ell_x) \qquad \text{(Frozen-in Model)} \tag{12a}$$

$$\lim_{\substack{C_{xt} \to 0 \\ C_{yt} \to 0}} S_D(\omega_D) = \sqrt{\pi} \, \tau_0 \exp\left[- \frac{\tau_0^2 \omega_D^2}{4} \right] \qquad \text{(Turbulent Model)} \tag{12b}$$

where $\delta(\cdot)$ is the Dirac delta function. For the frozen-in model, the drift velocity of striations is chosen to be along the x-direction in the plane normal to the line-of-sight (i.e., C_{xt} is set to unity and C_{yt} is set to zero). Because this direction is also chosen to be the one with the minimum decorrelation distance, this choice minimizes the resulting decorrelation time.

2.2.1 Frequency Selective Bandwidth and ω_{coh}.

The frequency selective bandwidth f_0 is an important measure of the effects of scintillation on the propagation of wide bandwidth signals. This quantity is defined in terms of the standard deviation of the time-of-arrival jitter, σ_τ:

$$f_0 = \frac{1}{2\pi\sigma_\tau} \tag{13}$$

where

$$\sigma_\tau^2 = \langle \tau^2 \rangle - \langle \tau \rangle^2 \ . \tag{14}$$

These delay moments can be calculated directly from the angle-delay part of the GPSD using the equation

$$P_0 \langle \tau^n \rangle = \int_{-\infty}^{\infty} \frac{d^2 K_\perp}{(2\pi)^2} \int_{-\infty}^{\infty} d\tau \, \tau^n \, S_{K\tau}(K_\perp, \tau) \ . \tag{15}$$

It is easy to show that the mean signal power P_0 is equal to unity.

The first and second moments are easily obtained in closed form giving the relationship between coherence bandwidth and the frequency selective bandwidth:

$$\omega_{coh} = 2\pi f_0 \left[1 + \frac{1}{\alpha^2}\right]^{\frac{1}{2}} . \tag{16}$$

The $1+1/\alpha^2$ term in the expression for the coherence bandwidth represents the relative contributions to the time delay jitter from diffraction (1) and dispersion ($1/\alpha^2$). In the limit that α is large, the time delay jitter is determined by diffractive effects alone. This should be the case under strong-scattering conditions.

2.2.2 Angle-of-Arrival Fluctuations and ℓ_x and ℓ_y.

A key parameter in determining the severity of antenna filtering effects is the standard deviation σ_θ of the angle-of-arrival jitter of the electric field incident on the antenna aperture. Clearly for anisotropic scattering, when ℓ_x and ℓ_y are unequal, values of σ_θ for scattering about the x and y axes will differ. The angle-of-arrival jitter variance about the x-direction for small angle scattering (i.e., $\sin\theta \approx \theta$) is equal to

$$\sigma_{\theta x}^2 = \int_{-\infty}^{\infty} \frac{d^2 K_\perp}{(2\pi)^2} \int_{-\infty}^{\infty} d\tau \left[\frac{\lambda K_x}{2\pi}\right]^2 S_{K\tau}(K_\perp, \tau) . \tag{17}$$

A similar expression holds for the y-direction angle-of-arrival jitter variance. The standard deviations of the angle-of-arrival about the x- and y-axes are then given by

$$\sigma_{\theta x} = \frac{\lambda}{\sqrt{2}\pi \ell_x} \tag{18a}$$

$$\sigma_{\theta y} = \frac{\lambda}{\sqrt{2}\pi \ell_y} . \tag{18b}$$

For small-angle scattering to be a valid assumption, the larger of the two angular standard deviations must be small relative to one radian. Thus the minimum decorrelation distance must be greater than the RF wavelength.

2.2.3 Isotropic Examples.

When the penetration angle is zero and the propagation path is aligned with the striations in the ionosphere, the two decorrelation distances are equal ($\ell_x = \ell_y = \ell_0$). In this case the scattering is isotropic about the line-of-sight, and the angle-delay part of the GPSD is one-dimensional:

$$S_{K\tau}(K, \tau) = \int_{-\infty}^{\infty} S_{K\tau}(K_x{=}K, K_y, \tau) \frac{dK_y}{2\pi} . \tag{19}$$

A three-dimensional plot of the angle-delay part of the GPSD for isotropic scattering is shown in Figure 2. This plot shows the mean received power as a function of normalized angle $K\ell_0$ and normalized delay $\omega_{coh}\tau$. The vertical axis is linear with arbitrary units. The value of α is 4 for this figure so consequently the quantity ω_{coh} is essentially equal to $2\pi f_0$.

It can be seen that the power arriving at large angles is also the power arriving at long delays. The power arriving at long delays thus has higher spatial frequency components than power arriving at short delays. When there is strong space-time correlation (i.e., when $(C_{xt}^2 + C_{yt}^2)^{1/2}$ is approximately equal to unity), these higher spatial frequency components correspond to higher Doppler frequency components. The signal arriving at long delays then varies more rapidly in time than the signal arriving at short delays.

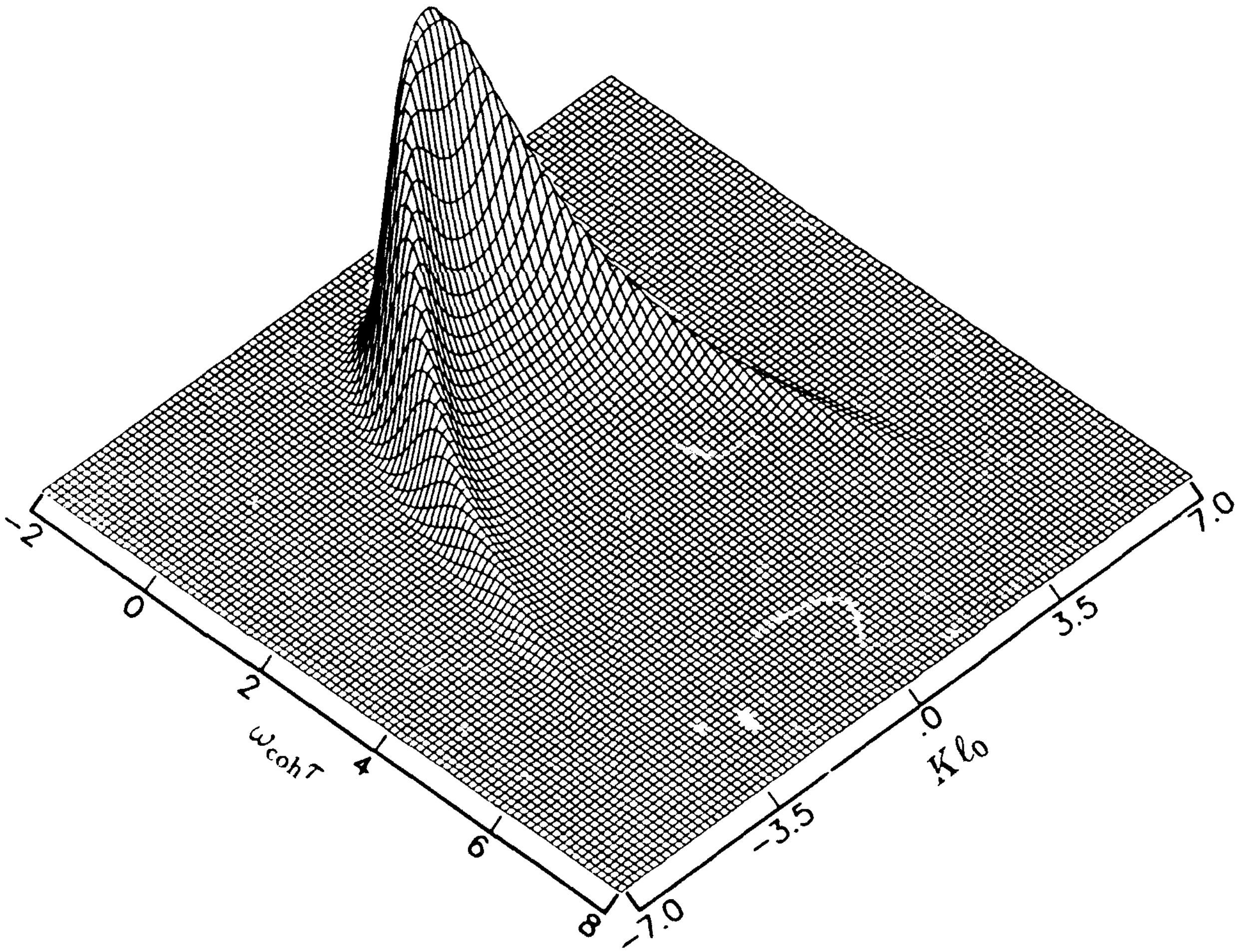

Figure 2. Angle-delay generalized power spectral density for isotropic scattering.

Another view of the GPSD can be obtained by considering the delay-Doppler scattering function:

$$S_{\tau D}(\tau,\omega_D) = \int_{-\infty}^{\infty} S(\boldsymbol{K}_\perp,\tau,\omega_D)\,\frac{d^2\boldsymbol{K}_\perp}{(2\pi)^2} \;. \tag{20}$$

A comparison of isotropic scattering functions for the frozen-in and turbulent models is shown in Figure 3. The frozen-in scattering function is just a reproduction of Figure 2 with normalized angle $K\ell_0$ replaced with normalized Doppler frequency $\tau_0\omega_D$. This is a consequence of the delta-function relationship between angle and Doppler frequency for the frozen-in model. For this model the signal at long delays has correspondingly large Doppler shifts, and a wing-like structure is seen in the scattering function. The turbulent model scattering function does not exhibit these Doppler wings because the Doppler spectrum is the same at all delays.

Both functions have exactly the same power density at each delay. The difference in appearance of the figures is because the turbulent model signal at long delay is more spread out in Doppler frequency and is therefore less obvious.

A progression of isotropic general model scattering functions is shown in Figure 4. For each of the plots in the figure C_{yt} is zero. The space-time correlation coefficient C_{xt} varies from 0.99 for the scattering function in the upper left to 0.7 for the scattering function in the lower right. The scattering function for C_{xt} equal to 0.99 is similar to that for the frozen-in model, and the scattering function for C_{xt} equal to 0.7 is similar to that for the turbulent model. For intermediate values of C_{xt}, the scattering functions still exhibit Doppler wings but the wings have broader Doppler spectra as C_{xt} decreases.

2.3 ANTENNA EFFECTS.

The GPSD of the signal at the output of an antenna, $S_A(\boldsymbol{K}_\perp,\tau,\omega_D)$, is given by the equation [*Dana*, 1991]

$$S_A(\boldsymbol{K}_\perp,\tau,\omega_D) = G(\boldsymbol{K}_\perp-\boldsymbol{K}_0)\,S(\boldsymbol{K}_\perp,\tau,\omega_D) \tag{21}$$

where $G(\boldsymbol{K}_\perp - \boldsymbol{K}_0)$ is the two dimensional beam profile of an antenna pointing in the direction $\boldsymbol{K}_0$. The effect of the antenna is to attenuate the received signal energy at angles outside the main beam of the antenna. If the antenna is pointed along the line-of-sight, the signal energy that is attenuated by the antenna is that with large angles-of-arrival. This is also the energy with large times-of-arrival or delay. The effect of the antenna will then be to reduce the received signal power, and to reduce the delay spread of that energy. As will be shown later, antenna beams pointed away from the line-of-sight suffer scattering loss but may not significantly reduce the delay spread of the received signal.

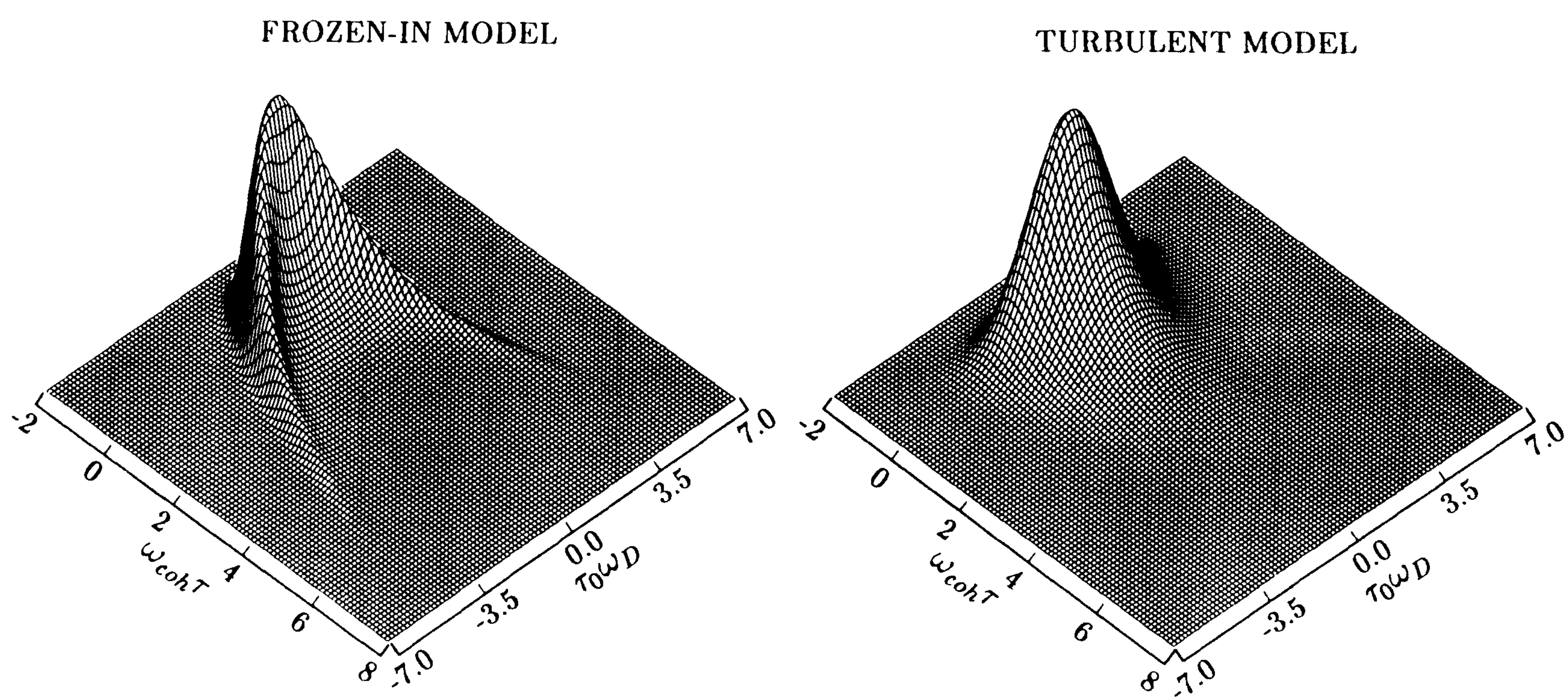

Figure 3. Comparison of isotropic frozen-in and turbulent model scattering functions.

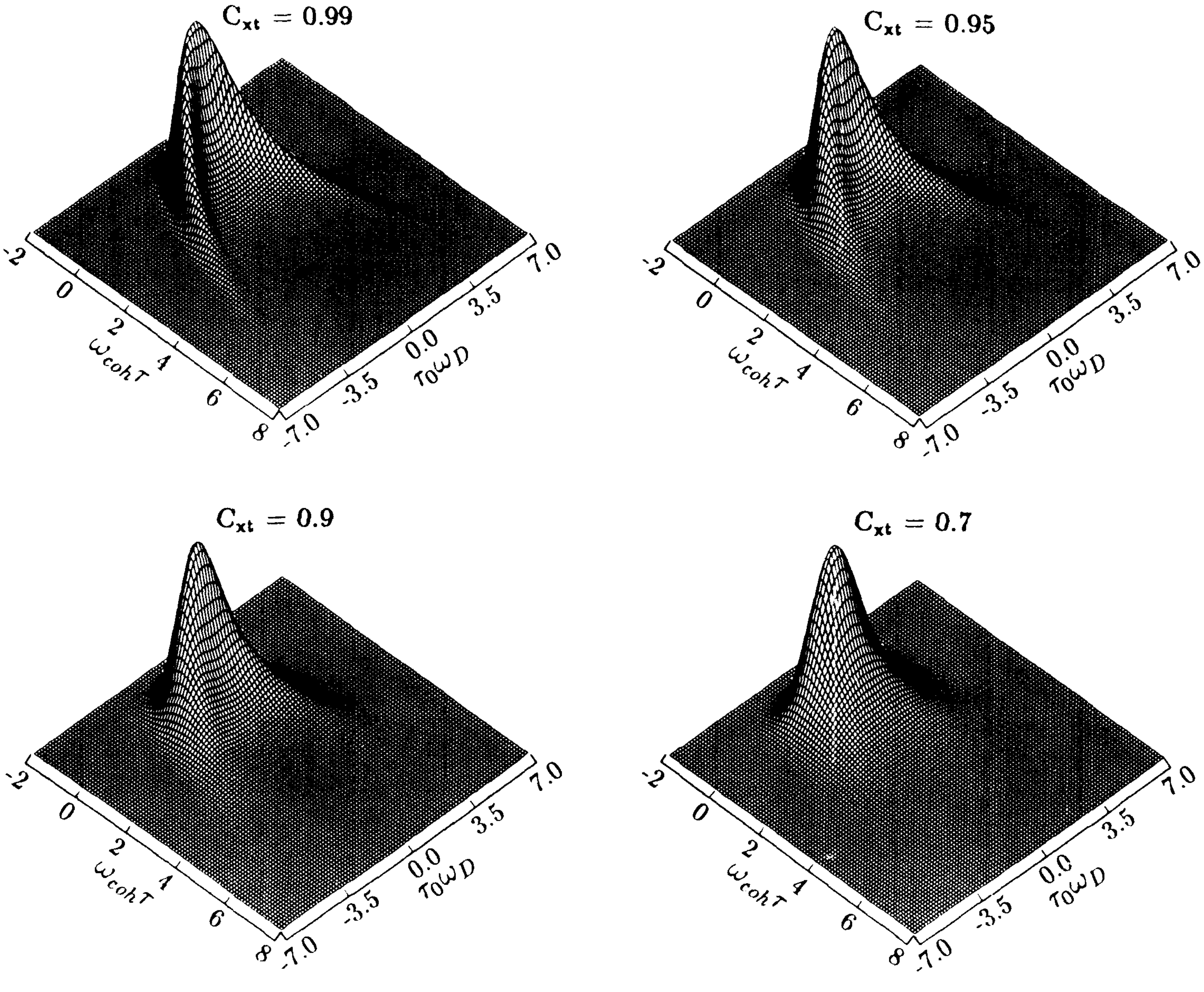

Figure 4. Isotropic general model scattering functions.

The antenna coordinate system is shown in Figure 5. It is assumed that the face of the antenna is in the x–y plane. The antenna beam is pointed away from the line-of-sight in the direction $\boldsymbol{K}_0$ defined by an elevation angle Θ_0 (measured from the line-of-sight) and an azimuth angle Φ_0. The rotation angle Ψ is the angle between the scattering $\hat{\boldsymbol{x}}$ axis and an antenna axis $\hat{\boldsymbol{u}}$. The antenna coordinate system $(\hat{\boldsymbol{u}},\hat{\boldsymbol{v}})$, where $\hat{\boldsymbol{v}}$ is in the x–y plane and is orthogonal to $\hat{\boldsymbol{u}}$, is chosen for convenience in describing the antenna beam profile. For example, if the antenna is rectangular, then the $(\hat{\boldsymbol{u}},\hat{\boldsymbol{v}})$ axes should be aligned with the sides of the aperture. In many cases the $\hat{\boldsymbol{u}}$ axis will be parallel to the local earth tangent plane.

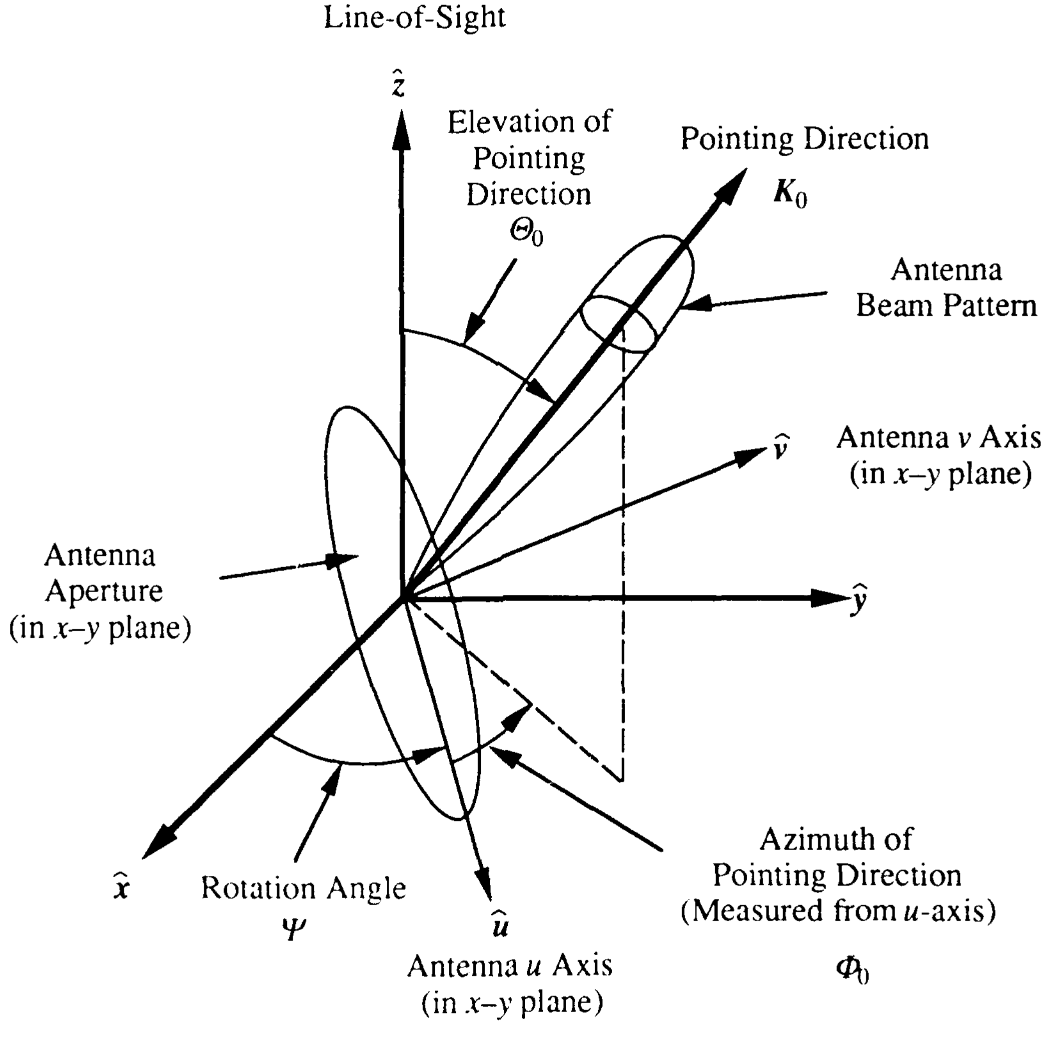

Figure 5. Antenna coordinate system.

The x- and y-components of the antenna pointing direction $\boldsymbol{K}_0$ in terms of the rotation, pointing, and azimuth angles are

$$K_{0x} = \frac{2\pi}{\lambda} \sin\Theta_0 \cos(\Psi + \Phi_0) \tag{22a}$$

$$K_{0y} = \frac{2\pi}{\lambda} \sin\Theta_0 \sin(\Psi + \Phi_0) \ . \tag{22b}$$

2.3.1 Antenna Descriptions.

The antenna beam profile is assumed to have a Gaussian shape and to be separable in the u–v coordinate system:

$$G(K_u,K_v) = G(K_u)G(K_u) = \exp\left[-\alpha_u^2 K_u^2\right] \exp\left[-\alpha_v^2 K_v^2\right] \tag{23}$$

where $\boldsymbol{K}_\perp=(K_u,K_v)$ in antenna coordinates. The peak gain, $G(0,0)$, has been set to unity because this value is usually included in the calculation of the mean received power. For either the u or the v direction, the antenna beam profile can also be written as

$$G(\theta) = \exp\left[-(\ln 2)\left(\frac{2\theta}{\theta_0}\right)^2\right] \tag{24}$$

where θ is the angle about either the u- or v-axis, and θ_0 is the 3 dB beamwidth [i.e., full width at half maximum, $G(\theta_0/2) = 1/2$]. Equating Equations 23 and 24 gives

$$\alpha_\xi^2 = \frac{(\ln 2)\ \lambda^2}{\pi^2 \theta_{0\xi}^2} \tag{25}$$

where $\theta_{0\xi}$ is the 3 dB beamwidth in either the $\xi = u$ or $\xi = v$ directions.

In the next subsections, the 3 dB beamwidths are related to antenna sizes for the special cases of uniformly weighted circular and rectangular apertures.

Uniformly Weighted Circular Apertures. In this case the well known form for the power beam profile is

$$G(\theta) = \frac{4J_1^2\,[\pi\,(D/\lambda)\sin\theta]}{[\pi\,(D/\lambda)\sin\theta]^2} \tag{26}$$

where $J_1(\cdot)$ is the first order Bessel function. The 3 dB (full width at half maximum) beamwidth, in terms of the aperture diameter D, is

$$\theta_0 = 1.02899\,\frac{\lambda}{D}\ \text{radians}\ . \tag{27}$$

If this beam profile is approximated by a Gaussian profile with the same 3 dB beamwidth, the coefficients that appear in Equation 23 are

$$\alpha_u^2 = \alpha_v^2 = \frac{(\ln 2)\, D^2}{(1.02899\pi)^2} \ . \tag{28}$$

Uniformly Weighted Rectangular Apertures. In this case the antenna beam profile in either the $\xi = u$ or $\xi = v$ direction has the familiar $\sin^2(x)/x^2$ form:

$$G_\xi(\theta) = \frac{\sin^2 [\pi\,(D_\xi/\lambda) \sin\theta]}{[\pi\,(D_\xi/\lambda) \sin\theta]^2} \tag{29}$$

where D_ξ is the length of the aperture. The 3 dB beamwidth is

$$\theta_{0\xi} = 0.885893 \frac{\lambda}{D_\xi} \text{ radians} \ , \tag{30}$$

and the coefficients that appear in Equation 23 are

$$\alpha_\xi^2 = \frac{(\ln 2)\, D_\xi^2}{(0.885893\pi)^2} \ . \tag{31}$$

A square aperture of size $D \times D$ has a smaller beamwidth than a circular aperture with diameter D because the square aperture has the larger area (D^2 versus $\pi D^2/4$).

2.3.2 Antenna Filtering Equations.

Filtering equations relate the statistics of the signal at the outputs of one or more antennas to the statistics of the signal incident on the antennas. Statistics that are considered in this subsection are mean power, frequency selective bandwidth, decorrelation distances, and decorrelation time of the signal out of an antenna. Gaussian beam patterns will be assumed for mathematical convenience. An elegant derivation of most of the equations given in this section may be found in *Frasier* [1990].

The severity of filtering effects is determined by the relative value of the standard deviation of the angle-of-arrival fluctuations, σ_θ, to the antenna beamwidths. When σ_θ is small compared to the antenna beamwidths, the signal arrives essentially at the peak of the beam pattern, if the pointing error is small, and filtering effects are small. However, if σ_θ is large compared to the beamwidths, much of the signal arrives at angles outside the main lobe of the antenna beam pattern, and filtering effects are large. Equivalently, large values of the ratio σ_θ/θ_0 correspond to situations where the decorrelation distance of the incident signal is small compared to the antenna size and the electric field incident on the aperture is no longer a plane wave. In this situation, induced voltages across the face of the aperture do not add coherently when summed together by the antenna with a resulting loss in signal power.

In the next subsections, expressions will be presented for the mean power, frequency selective bandwidth, decorrelation time, and decorrelation distances of the signal at the output of an antenna. Clearly the numerical value of these quantities must be independent of the choice of coordinate systems (e.g., x–y or u–v coordinates). *Frasier* [1990] has derived expression for these quantities that are coordinate-system independent. However, evaluation of the filtering equations requires a choice of coordinate systems, and the x–y system will be used here.

Before presenting the filtering equations, it is convenient to define some quantities that are common to these equations:

$$Q_x = 1 + \frac{4\alpha_u^2}{\ell_x^2}\cos^2\Psi + \frac{4\alpha_v^2}{\ell_x^2}\sin^2\Psi \tag{32a}$$

$$Q_y = 1 + \frac{4\alpha_u^2}{\ell_y^2}\sin^2\Psi + \frac{4\alpha_v^2}{\ell_y^2}\cos^2\Psi \tag{32b}$$

$$Q_{xy} = \frac{4(\alpha_u^2 - \alpha_v^2)}{\ell_x \ell_y}\cos\Psi\sin\Psi \tag{32c}$$

$$Q_0 = \sqrt{Q_x Q_y - Q_{xy}^2} \; . \tag{32d}$$

Scattering Loss. The mean power P_A of the signal out of an antenna is calculated using the expression:

$$P_A = \int_{-\infty}^{\infty} \frac{d^2K_\perp}{(2\pi)^2} \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} \frac{d\omega_D}{2\pi} S_A(K_\perp,\tau,\omega_D) \tag{33}$$

which gives the result

$$P_A = \frac{1}{Q_0}\exp\left[-\left(1 - \frac{Q_y}{Q_0^2}\right)\frac{K_{0x}^2\ell_x^2}{4} - \left(1 - \frac{Q_x}{Q_0^2}\right)\frac{K_{0y}^2\ell_y^2}{4} - \frac{Q_{xy}}{2Q_0^2}K_{0x}\ell_x K_{0y}\ell_y\right] . \tag{34}$$

The first term in this expression, $1/Q_0$, is just the mean received power when the antenna is pointed along the line-of-sight.

The scattering loss of the antenna in decibels is just

$$L\,(\mathrm{dB}) = -10\log_{10}(P_A) \; . \tag{35}$$

As defined here, scattering loss includes both the losses in the mean received power due to scintillation and that due to pointing the antenna beam away from the line-of-sight.

Frequency Selective Bandwidth. The frequency selective bandwidth is defined in terms of the standard deviation of the time-of-arrival (i.e., delay) jitter of the signal. At the output of an antenna, the time delay moments of the received signal are given by

$$P_A\langle \tau^n \rangle = \int_{-\infty}^{\infty} \frac{d^2K_\perp}{(2\pi)^2} \int_{-\infty}^{\infty} d\tau\, \tau^n \int_{-\infty}^{\infty} \frac{d\omega_D}{2\pi} S_A(K_\perp,\tau,\omega_D) \ . \tag{36}$$

Straightforward evaluation of the indicated integrals for n equal to 1 and 2 is indeed a formidable algebraic task. However, *Frasier* [1990] gives surprisingly simple-appearing expressions for σ_τ and the antenna-filtered value of the frequency selective bandwidth, f_A, that are independent of the choice of coordinate system. In terms of the notation used in this report, the expression for f_A is somewhat more complicated.

The antenna filtered frequency selective bandwidth is

$$\frac{f_A}{f_0} = \frac{\left[\dfrac{\ell_x^4 + \ell_y^4}{\ell_x^2\ell_y^2}\right]^{\frac{1}{2}} Q_0^2}{\left[\dfrac{\ell_x^2 Q_x^2}{\ell_y^2} + \dfrac{\ell_y^2 Q_y^2}{\ell_x^2} + 2Q_{xy}^2 + Q_P\right]^{\frac{1}{2}}} \tag{37}$$

where the term Q_P in the denominator gives the effects of antenna pointing. This pointing angle term may be written as

$$Q_P = Q_{Px}K_{0x}^2\ell_x^2 + Q_{Py}K_{0y}^2\ell_y^2 + Q_{Pxy}K_{0x}\ell_x^2K_{0y}\ell_y^2 \tag{38a}$$

$$Q_{Px} = 2Q_{xy}^2(1 - Q_x) + \frac{Q_y\ell_y^2(Q_0^2 - Q_y)^2}{Q_0^2\ell_x^2} + \frac{Q_xQ_{xy}^2\ell_x^2}{Q_0^2\ell_y^2} \tag{38b}$$

$$Q_{Py} = 2Q_{xy}^2(1 - Q_y) + \frac{Q_x\ell_x^2(Q_0^2 - Q_x)^2}{Q_0^2\ell_y^2} + \frac{Q_yQ_{xy}^2\ell_y^2}{Q_0^2\ell_x^2} \tag{38c}$$

$$Q_{Pxy} = Q_{xy}\left[(Q_x - 1)(1 - Q_y) - Q_{xy}^2 + \frac{Q_x\ell_x^2(Q_0^2 - Q_x)}{Q_0^2\ell_y^2} + \frac{Q_y\ell_y^2(Q_0^2 - Q_y)}{Q_0^2\ell_x^2}\right] \tag{38d}$$

Decorrelation Time. The temporal coherence function of the signal out of an antenna is given by the expression

$$P_A \Gamma_A(t) = \int_{-\infty}^{\infty} \frac{d^2 \boldsymbol{K}_\perp}{(2\pi)^2} \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} \frac{d\omega_D}{2\pi} \exp\left(-i\omega_D t\right) S_A(\boldsymbol{K}_\perp, \tau, \omega_D) \ . \tag{39}$$

In general, the temporal coherence function is complex when the pointing angle is non-zero because antenna pointing results in a mean Doppler shift of the output signal. The antenna-filtered decorrelation time τ_A is then calculated by finding the $1/e$ point of $|\Gamma_A(t)|$ with the result

$$\frac{\tau_A}{\tau_0} = \left[1 - C_{xt}^2 - C_{yt}^2 + \frac{C_{xt}^2 Q_y + C_{yt}^2 Q_x - 2C_{xt}C_{yt}Q_{xy}}{Q_0^2} \right]^{\frac{1}{2}} \ . \tag{40}$$

The frozen-in ($C_{xt} = 1$, $C_{yt} = 0$) and turbulent ($C_{xt} = C_{yt} = 0$) limits of Equation 40 are

$$\frac{\tau_A}{\tau_0} = \frac{Q_0}{\sqrt{Q_y}} \qquad \text{(Frozen-in Model)} \tag{41a}$$

$$\frac{\tau_A}{\tau_0} = 1 \qquad \text{(Turbulent Model)} \ . \tag{41b}$$

It is interesting that the antenna-filtered decorrelation time is not explicitly dependent on pointing angle, but does depend on the ratio of the antenna beamwidth and the standard deviation of the angle-of-arrival jitter through the Q factors. Of course as beam is scanned away from the antenna boresight, the effective aperture size will decrease thereby broadening the beamwidth and implicitly changing the value of the filtered decorrelation time.

The mean Doppler shift ω_A due to antenna pointing is given by the imaginary part of $\Gamma_A(t)$ and is

$$\omega_A = \frac{[C_{xt}(Q_0^2 - Q_y) - C_{yt}Q_{xy}]K_{0x}\ell_x - [C_{yt}(Q_0^2 - Q_x) - C_{xt}Q_{xy}]K_{0y}\ell_y}{\tau_0 Q_0^2} \ . \tag{42}$$

This expression is clearly equal to zero for the turbulent model. For non-zero values of the space-time correlation coefficients, the mean Doppler shift is proportional to the components of the pointing vector.

Decorrelation Distances. The x-direction decorrelation distance of the signal out of the antenna is given by the $1/e$ point of $\Gamma_A(x)$, where

$$P_A \Gamma_A(x) = \int_{-\infty}^{\infty} \frac{d^2 K_\perp}{(2\pi)^2} \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} \frac{d\omega_D}{2\pi} \exp\,(iK_x x)\, S_A(K_\perp, \tau, \omega_D) \ . \tag{43}$$

A similar expression holds for $\Gamma_A(y)$.

These two expressions give the following results for the decorrelation distances ℓ_{Ax} and ℓ_{Ay} of the signal at the antenna output:

$$\frac{\ell_{Ax}}{\ell_x} = \frac{Q_0}{\sqrt{Q_y}} \tag{44a}$$

$$\frac{\ell_{Ay}}{\ell_y} = \frac{Q_0}{\sqrt{Q_x}} \ . \tag{44b}$$

The interpretation of these quantities is that ℓ_{Ax} and ℓ_{Ay} are the distances in the x- or y-direction, respectively, that the antenna must be instantaneously displaced for the normalized cross correlation of the output signal to have a value of $1/e$. As was true for the antenna-filtered decorrelation time, these quantities do not explicitly depend on pointing angle.

2.3.3 Isotropic Example.

An isotropic example is presented in this section to illustrate some of the effects of the antenna on the parameters discussed above. For this example, both the angular scattering and the antenna beam pattern will be assumed to be isotropic, so

$$\ell_x = \ell_y = \ell_0 \tag{45}$$

and

$$\alpha_u^2 = \alpha_v^2 = \frac{(\ln 2)\,\lambda^2}{\pi^2 \theta_0^2} \ . \tag{46}$$

The effect of antenna filtering is illustrated in Figure 6, which shows four plots of the angle-delay part of the GPSD at the outputs of uniformly-weighted circular antennas for isotropic scattering. The antennas in this example are all pointed along the line-of-sight. The upper left plot is the same as in Figure 2 and is for a point antenna ($D << \ell_0$), for which there is no antenna filtering. The other three plots are for cases where the ratio of antenna diameter to decorrelation distance, D/ℓ_0, is equal to 1, 2, and 4. As the antenna size increases for a given value of ℓ_0 or, equivalently, as the decorrelation distance decreases for a given antenna diameter D, more of the energy arriving at large scattering angles is filtered out.

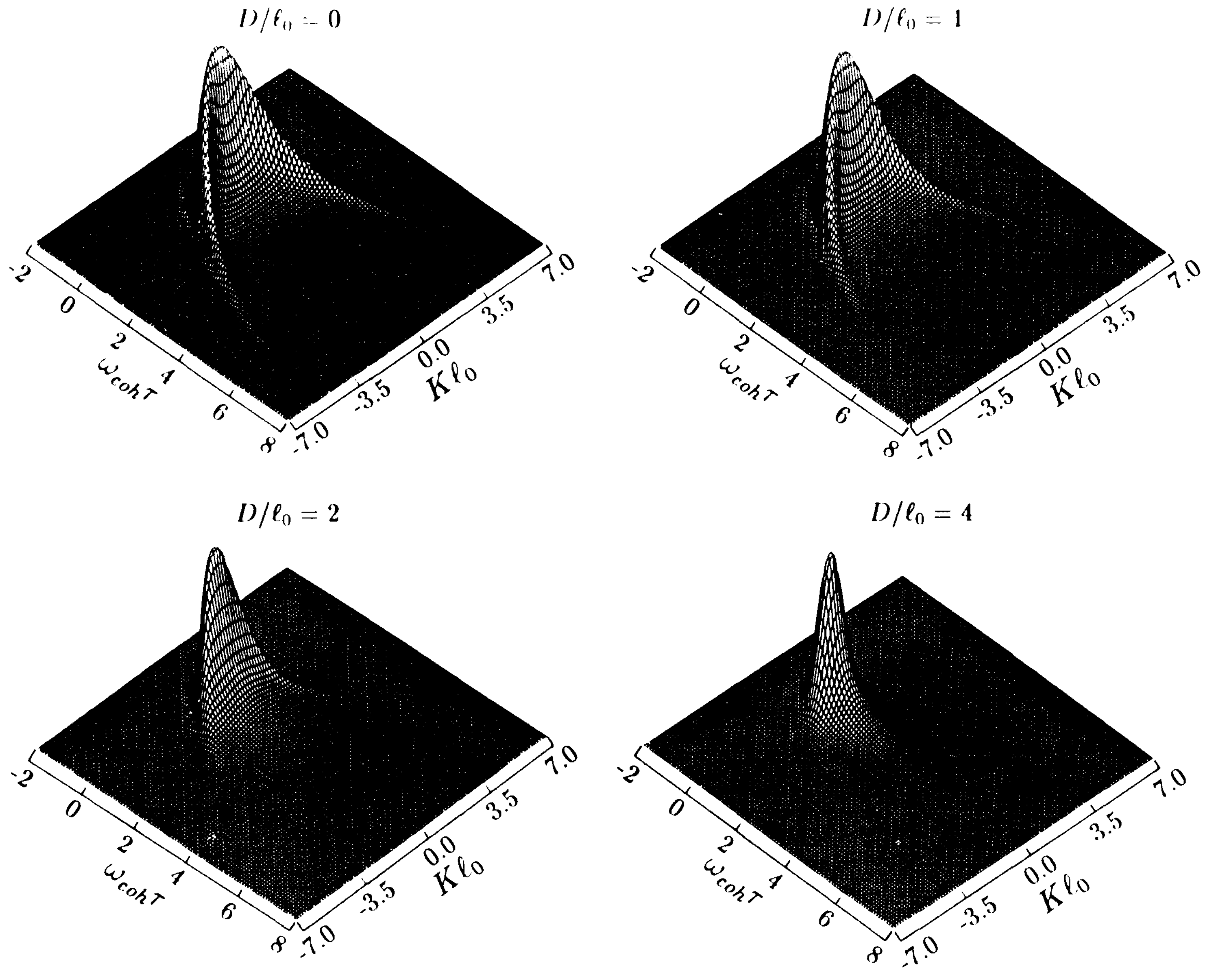

Figure 6. Generalized power spectra for isotropic scattering at the outputs of uniformly-weighted circular antennas.

Antenna filtering has two major effects, one of which is generally bad and the other potentially good in terms of system performance. The bad effect is scattering loss. Energy that arrives at large angles is not captured by the relatively narrow beam of a large aperture antenna. An equivalent statement is that spatial decorrelation of the received electric field across the antenna aperture causes a reduction in the effective gain of the antenna. The magnitude of the scattering loss is a function of the ratio of the antenna diameter to decorrelation distance of the incident wave and a function of the pointing angle of the antenna beam away from the line-of-sight.

The potentially beneficial effect of antenna filtering for beams pointed along the line-of-sight is that the energy which is filtered out is that which arrives at relatively large delays. It is this delayed energy that causes most of the intersymbol interference in the detection and demodulation of wide bandwidth signals. Stated another way, because the antenna filters out much of the delayed energy, and since the frequency selective bandwidth is an inverse measure of the signal delay spread, the filtering increases the frequency selective bandwidth of the output signal relative that at the antenna input.

To simplify the filtering equations and without further loss of generality, it can be assumed that the antenna is pointed away from the line-of-sight in the x-direction and that the rotation angle is zero. The components of the pointing vector are then given by

$$K_{0x} = \frac{2\pi}{\lambda} \sin \Theta_0 \tag{47a}$$

$$K_{0y} = 0 \ . \tag{47b}$$

With the assumption of isotropic scattering and antennas, the Q factors become

$$Q = Q_x = Q_y = 1 + \frac{4(\ln 2)}{\pi^2 \theta_0^2} \frac{\lambda^2}{\ell_0^2} \tag{48a}$$

$$Q_{xy} = 0 \ . \tag{48b}$$

At this point it is convenient to write the antenna beamwidth in terms of the antenna size to eliminate the explicit dependence of the Q factors on carrier wavelength. Thus let

$$\theta_0 = a_0 \frac{\lambda}{D} \tag{49}$$

where, for uniformly-weighted antennas,

$$a_0 = \begin{cases} 1.02899 & \text{Circular antenna} \\ 0.885893 & \text{Square antenna} \end{cases} \ . \tag{50}$$

The Q factor can then be written as

$$Q = \begin{cases} 1 + 0.265 \dfrac{D^2}{\ell_0^2} & \text{Circular antenna} \\ 1 + 0.358 \dfrac{D^2}{\ell_0^2} & \text{Square antenna} \end{cases} \quad . \tag{51}$$

The mean received power for isotropic scattering and an isotropic antenna is

$$P_A = \frac{1}{Q} \exp\left[- \frac{4 \ln 2}{Q} \frac{\Theta_0^2}{\theta_0^2} \right] \tag{52}$$

when the pointing angle is assumed to be small compared to one radian so sin Θ_0 is approximately equal to Θ_0. For a given value of ℓ_0/D (i.e., for a fixed value of Q) the effect of antenna pointing, as expected, is to monotonically decrease the mean received power as the antenna beam is pointed farther away from the line-of-sight. In the limit that the decorrelation distance is much smaller than the antenna diameter, the exponent in Equation 52 approaches zero and the mean received power is approximately equal to $1/Q$ independent of pointing angle. In other words, when the angular scattering is large compared to the antenna beamwidth and pointing angle, the mean power is insensitive to the location of the beam peak relative to the line-of-sight.

The scattering loss in decibels for a uniformly-weighted circular antenna is plotted in Figure 7 versus the ratio ℓ_0/D when the pointing angle of the antenna beam is 0, $\theta_0/2$ and θ_0. It is assumed for this example that the beamwidth remains constant as the beam in pointed away from the line-of-sight. When the decorrelation distance is large compared to the antenna diameter, the curves approach the line-of-sight loss of beam (0 dB for a pointing angle equal to zero, 3 dB for a pointing angle equal to one-half beamwidth, and 12 dB for a pointing angle equal to a full beamwidth). When ℓ_0/D is small, the angle-of-arrival spread of the incident signal is large compared to the beamwidth, so, as mentioned above, the scattering loss is insensitive to pointing angle as long as the beam remains within a few beamwidths of the line-of-sight. Between these two limits the scattering loss of a beam pointed away from the line-of-sight may actually decrease with decreasing ℓ_0/D as the signal is scattered away from the line-of-sight and into the beam.

The equation for the ratio of the filtered to unfiltered frequency selective bandwidth for this isotropic example is

$$\frac{f_A}{f_0} = \left\{ \frac{Q^3}{Q + 8 \ln 2 (Q - 1) \left[\dfrac{\Theta_0}{\theta_0} \right]^2} \right\}^{\frac{1}{2}} . \tag{53}$$

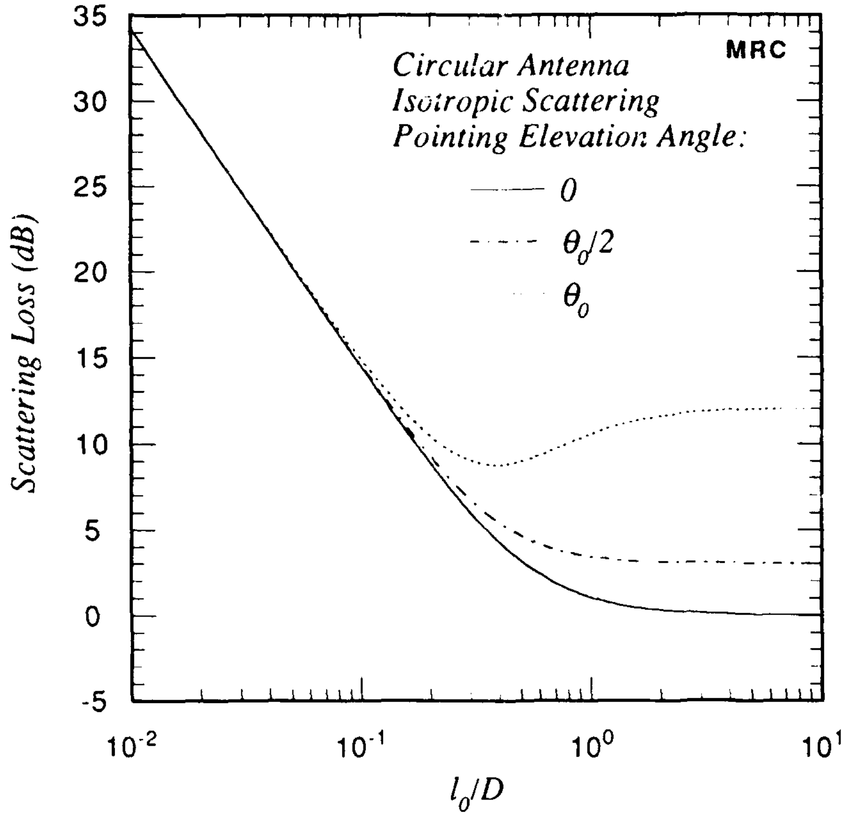

Figure 7. Scattering loss for a uniformly-weighted circular antenna and isotropic scattering.

It is noteworthy that when the pointing angle is zero the ratio f_A/f_0 is equal to Q, which is also equal to the scattering loss in this case. Figure 8 shows plots of the ratio f_A/f_0 as a function of the ratio ℓ_0/D for a uniformly-weighted circular antenna and three pointing angles.

The potentially beneficial effect of antenna filtering for a beam pointed along the line-of-sight is that the signal which is filtered out is that which arrives at relatively large delays. Clearly the situation is different if the antenna is pointed away from the line-of-sight. The ratio f_A/f_0 is less than unity for non-zero pointing angles and values of ℓ_0/D between about 0.1 and 1.0. This implies that the standard deviation of the delay jitter is increased by the antenna. This *does not* imply however that the antenna is somehow creating more signal power at long delays than is incident on the antenna, as measured by f_0. Rather the antenna pointed away from the line-of-sight is weighting the power at long delays more than that at short delays with the peak weighting occurring at the delay that corresponds to the pointing angle. An increased value of time delay jitter standard deviation results. Thus intersymbol interference effects may increase in the output as an antenna beam is pointed away from the line-of-sight.

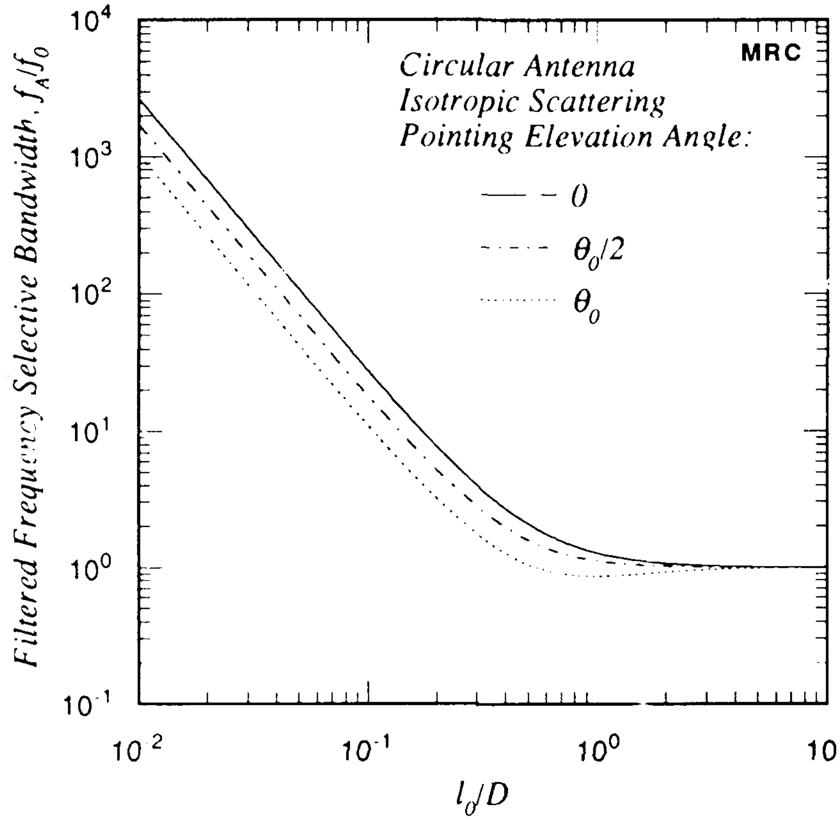

Figure 8. Filtered frequency selective bandwidth for a uniformly-weighted circular antenna and isotropic scattering.

Antenna filtering also affects the decorrelation time of the received signal. For isotropic scattering and an isotropic antenna, the equation for the ratio τ_A/τ_0 reduces to

$$\frac{\tau_A}{\tau_0} = \left[\frac{Q}{Q + (C_{xt}^2 + C_{yt}^2)(1 - Q)}\right]^{\frac{1}{2}} . \tag{54}$$

This equation is plotted in Figure 9 versus the ratio ℓ_0/D for various values of

$$C = \sqrt{C_{xt}^2 + C_{yt}^2} . \tag{55}$$

As discussed above, the antenna-filtered value of decorrelation time is not explicitly dependent on pointing angle. It does, however, depend strongly on the model for the temporal fluctuations.

For the turbulent model ($C = 0$) the Doppler spectrum is independent of angle, so the signal arriving at all angles has the same decorrelation time. Consequently the filtered value of decorrelation time is equal to that of the incident signal (i.e., $\tau_A/\tau_0 = 1$ for this model). For larger values of C there is coupling between the angular and Doppler spectra of the incident signal. The effect of an antenna is to narrow the angular spectrum of the received signal, as shown in Figure 7. Thus when C is greater than zero the antenna also narrows the Doppler spectrum of the received signal. Because the decorrelation time of the signal is an inverse measure of the width of the Doppler spectrum, the decorrelation time of the signal out of the antenna increases as antenna filtering increases. For the frozen-in case, C is equal to unity and $\tau_A/\tau_0 = Q^{1/2}$. This expression gives the upper limit of the antenna-filtered decorrelation time.

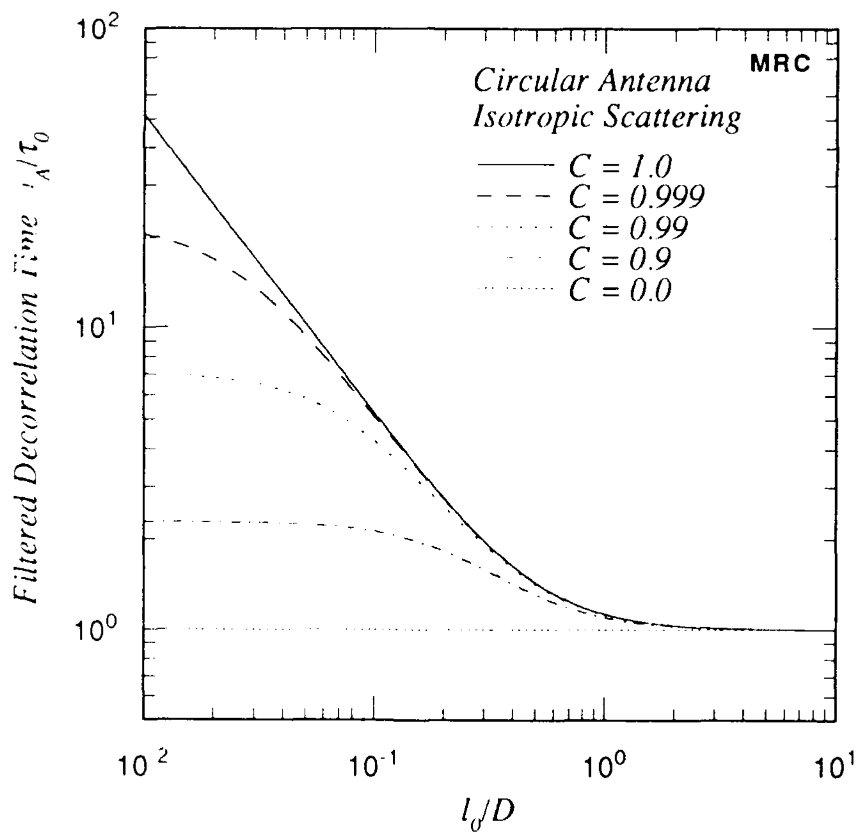

Figure 9. Filtered decorrelation time for a uniformly-weighted circular antenna and isotropic scattering.

The corresponding mean Doppler frequency shift is

$$\omega_A = \frac{2\pi a_0 C_{xt}(Q-1)}{\tau_0 Q} \frac{\Theta_0}{\theta_0} \frac{\ell_0}{D} \ . \tag{56}$$

The normalized mean Doppler shift due to antenna pointing, $\tau_0\omega_A/C_{xt}$, is plotted in Figure 10 versus the ratio ℓ_0/D. The largest mean Doppler frequency shift occurs when ℓ_0/D is approximately equal to 0.5.

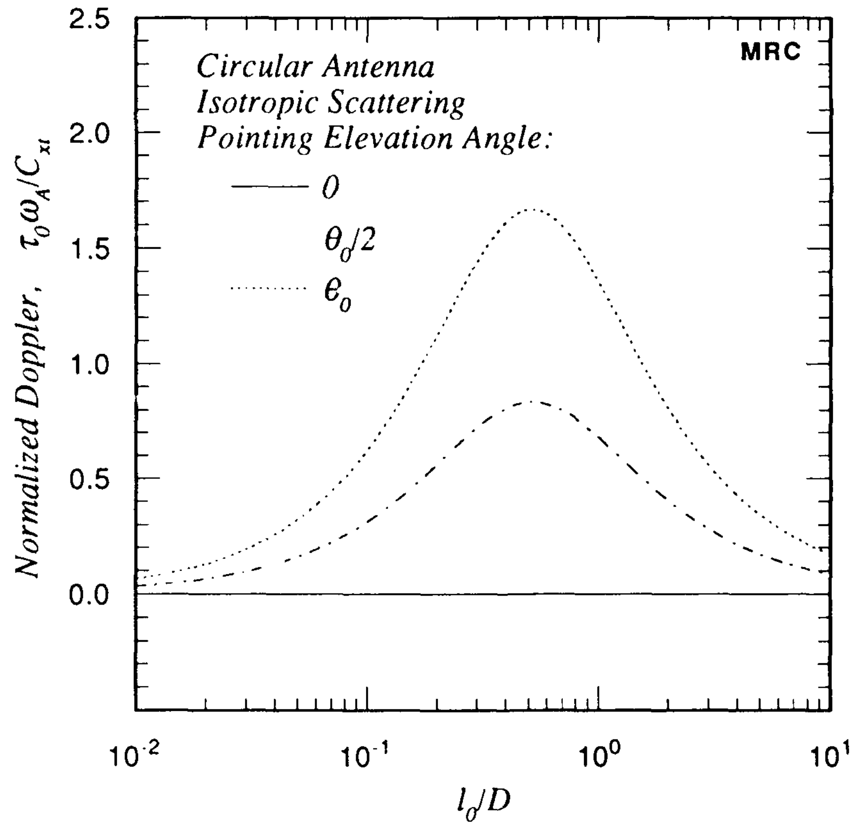

Figure 10. Normalized mean Doppler shift due to antenna pointing for a uniformly-weighted circular antenna and isotropic scattering.

SECTION 3
CHANNEL SIMULATION

A statistical channel simulation technique is described in this section that is used to generate realizations of the impulse response functions at the outputs of multiple antennas with spatial and temporal correlation properties given by the mutual coherence function and with Rayleigh amplitude statistics. Realizations generated with this technique represent only the diffractive part of the received voltage. They are valid only under strong-scattering conditions where the GPSD is valid and where Rayleigh statistics apply. Under these conditions, however, they represent a solution of Maxwell's equations for propagation of RF waves through randomly structured ionization.

The channel simulation technique for the General Model that is described here was developed by Dr. Leon A. Wittwer of the Defense Nuclear Agency (DNA), and has been implemented in the channel model Fortran code ACIRF written by the author and Dr. Wittwer.

The basic formalism used to generate statistical realizations of the channel impulse response function without antenna effects explicitly included was first developed by *Wittwer* [1980] for isotropic irregularities. *Knepp and Wittwer* [1984] extended the technique to the case of elongated irregularities. (The elongated case corresponds to a 90° penetration angle and to an infinite axial ratio.) The channel simulation technique was further generalized by *Dana* [1986] to include the effects of general anisotropic scattering and multiple antennas.

3.1 IMPULSE RESPONSE FUNCTION.

A key assumption used to generate realizations of the channel impulse response function is that the channel is statistically stationary (in the wide sense) in space, frequency, and time. As a consequence, the impulse response function is delta correlated in angle, delay, and Doppler frequency. This allows the impulse response function to be generated from white Gaussian noise in the angle, delay, and Doppler frequency domains and then Fourier transformed to the space and time domains. If necessary for a particular application, the Fourier transform from delay to frequency may also be performed to obtain the channel transfer function.

The impulse response function at the output of an aperture antenna located at ρ_0 and pointed in direction K_0 is given by the expression [*Dana*, 1991]

$$h_A(\rho_0,\tau,t) = \int_{-\infty}^{\infty} h(\rho,\tau,t)\, A(\rho-\rho_0) \exp\left[iK_0\cdot(\rho-\rho_0)\right] d^2\rho \quad . \tag{57}$$

This equation represents the spatial convolution of the aperture weighting function $A(\rho)$ and the impulse response function $h(\rho,\tau,t)$ of the signal incident on the face of the aperture.

The channel simulation technique depends on writing $h_A(\rho,\tau,t)$ in terms of Fourier transforms from angle to space and from Doppler frequency to time. To show that the equation can be written in this form, consider writing $h(\rho,\tau,t)$ in terms of its Fourier transform, $\hat{h}(\boldsymbol{K}_\perp,\tau,\omega_D)$:

$$h(\rho,\tau,t) = \int_{-\infty}^{\infty} \frac{d^2\boldsymbol{K}_\perp}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{d\omega_D}{2\pi} \exp\left[i(\boldsymbol{K}_\perp \cdot \rho - \omega_D t)\right] \hat{h}(\boldsymbol{K}_\perp,\tau,\omega_D) \ . \tag{58}$$

The aperture weighting function $A(\rho)$ can also be written in terms of its Fourier transform, the antenna voltage pattern $g(\boldsymbol{K}_\perp)$:

$$A(\rho) = \int_{-\infty}^{\infty} g(\boldsymbol{K}_\perp) \exp\left(i\boldsymbol{K}_\perp \cdot \rho\right) \frac{d^2\boldsymbol{K}_\perp}{(2\pi)^2} \ . \tag{59}$$

Substituting these expressions into Equation 57 and doing the usual change in the order of integration to produce a delta function results in

$$h_A(\rho_0,\tau,t) = \int_{-\infty}^{\infty} \frac{d^2\boldsymbol{K}_\perp}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{d\omega_D}{2\pi} \exp\left[i(\boldsymbol{K}_\perp \cdot \rho_0 - \omega_D t)\right] g(\boldsymbol{K}_\perp - \boldsymbol{K}_0) \ \hat{h}(\boldsymbol{K}_\perp,\tau,\omega_D) \ . \tag{60}$$

This equation is used to generate realizations of the impulse response function by first generating random samples of $\hat{h}(\boldsymbol{K}_\perp,\tau,\omega_D)$ and then performing the indicated Fourier transforms. Because of the assumption of a statistically stationary channel, $\hat{h}(\boldsymbol{K}_\perp,\tau,\omega_D)$ must be delta correlated in angle, delay, and Doppler frequency:

$$\langle \hat{h}(\boldsymbol{K}_\perp,\tau,\omega_D)\, \hat{h}^*(\boldsymbol{K}'_\perp,\tau',\omega'_D)\rangle = S(\boldsymbol{K}_\perp,\tau,\omega_D)\, \delta(\boldsymbol{K}_\perp - \boldsymbol{K}'_\perp)\, \delta(\tau-\tau')\, \delta(\omega_D - \omega'_D) \ . \tag{61}$$

The first-order amplitude statistics of the complex quantity $h(\rho,\tau,t)$ are Rayleigh, a consequence of the central limit theorem. That is, $h(\rho,\tau,t)$ represents the summation of many scattered waves travelling in slightly different directions. Thus the two orthogonal components of $h(\rho,\tau,t)$ (either the in-phase and quadrature-phase components or, in the notation used in this report, the real and imaginary parts) are independent, zero mean, normally-distributed random variables. Consequently the resulting amplitude is Rayleigh distributed, and the resulting phase is uniformly distributed. Equation 60 indicates that $h_A(\rho,\tau,t)$ is the summation or integration of weighted values of $h(\rho,\tau,t)$ and is therefore also Rayleigh distributed (i.e., the sum of normally-distributed random variables is itself normally distributed). Strictly speaking the statistics of $\hat{h}(\boldsymbol{K}_\perp,\tau,\omega_D)$ could be almost anything that obeys Equation 61, and the central limit theorem could be invoked to argue that $h(\rho,\tau,t)$ and $h_A(\rho,\tau,t)$ are zero-mean, normally-distributed complex quantities. Indeed multiple phase screen techniques can be used to generate Rayleigh-distributed realizations of $h(\rho,\tau,t)$ starting

with just random phase perturbations of an electric field. However, such faith in the central limit theorem is not necessary if $\hat{h}(K_\perp,\tau,\omega_D)$ starts out as a zero-mean, normally-distributed complex quantity. This allows many fewer points to be used in performing discrete Fourier transforms from angle to space than would otherwise be required to guarantee Rayleigh statistics.

3.2 GENERATION OF REALIZATIONS.

The first step necessary to generate realizations of the impulse response function is evaluation of the GPSD on an angular K_x-K_y grid. Once the power in the angular grid cells has been obtained, these quantities are used to construct random samples of the angular spectrum of the signal. A delta-function relationship between angle and delay is used to relate an annulus in the K_x-K_y grid to specific delay bins, and translation properties of the GPSD are used to relate specific angles to Doppler bins. The random angular spectrum is then multiplied by the antenna beam pattern, and a two-dimensional discrete Fourier transform (DFT) is performed to the antenna phase center location. A final Fourier transform from the Doppler frequency domain produces the impulse response as a function of time and delay. Examples of such realizations are presented in Section 4.

Before launching into a detailed discussion of the channel simulation technique, it is useful to consider some properties of the GPSD.

3.2.1 GPSD Used in Channel Modeling.

Under the strong-scattering conditions in the ionosphere that cause signal scintillation at radio frequencies, diffractive effects dominate dispersive effects. In this case the value of α in the GPSD is large. In the limit that α approaches infinity, the angle-delay part of the GPSD becomes

$$S_{K\tau}(K_\perp,\tau) = \pi \ell_x \ell_y\, \omega_{coh} \exp\left[-\frac{K_x^2\ell_x^2}{4} - \frac{K_y^2\ell_y^2}{4}\right] \times \delta\left[\omega_{coh}\tau - \frac{\Lambda_y(K_x^2 + K_y^2)\ell_x^2}{4}\right]. \tag{62}$$

The range of delay in this equation is from 0 to $+\infty$ because the second term in the delta function is positive.

This geometric optics limit results in a delta-function relationship between angle and delay. The delta function in this equation can be used to associate the signal with a particular angle-of-arrival to a specific delay. This association is used to develop efficient channel simulation techniques.

Once the diffraction limit has been taken, the GPSD used in channel modeling can be integrated over delay producing an angle-Doppler form, denoted S_{KD}. After some rearrangement of terms, this form of the GPSD is

$$S_{KD}(K_\perp,\omega_D) = \int_0^\infty S(K_\perp,\tau,\omega_D)\,d\tau \tag{63}$$

$$= \frac{\pi^{3/2}\ell_x \ell_y \tau_0}{\sqrt{1 - C_{xt}^2 - C_{yt}^2}} \exp\left[-\frac{\tau_0^2\omega_D^2}{4}\right]$$

$$\times \exp\left[-\frac{(K_x\ell_x - C_{xt}\tau_0\omega_D)^2(1 - C_{yt}^2) + (K_y\ell_y - C_{yt}\tau_0\omega_D)^2(1 - C_{xt}^2)}{4(1 - C_{xt}^2 - C_{yt}^2)}\right]$$

$$\times \exp\left[-\frac{2C_{xt}C_{yt}(K_x\ell_x - C_{xt}\tau_0\omega_D)(K_y\ell_y - C_{yt}\tau_0\omega_D)}{4(1 - C_{xt}^2 - C_{yt}^2)}\right] .$$

This form of the GPSD shows, aside from the leading exponential factor, that the effect of Doppler frequency is to shift the GPSD in $K_x - K_y$ space. Thus it is possible to evaluate Equation 63 for zero Doppler frequency and then to shift the zero point of $K_x - K_y$ coordinate system to obtain the GPSD for non-zero Doppler frequencies.

Equation 63 does present a problem due to its $K_x - K_y$ cross term. This term makes the evaluation of the signal power in each $K_x - K_y$ grid cell computationally difficult and therefore time consuming. Of course a simple rotation can be used to eliminate this term. Thus consider a new coordinate system, $K_p - K_q$, where

$$K_p = K_x \cos\vartheta + K_y \sin\vartheta \tag{64a}$$

$$K_q = -K_x \sin\vartheta + K_y \cos\vartheta \ . \tag{64b}$$

The rotation angle ϑ between the $K_x - K_y$ and $K_p - K_q$ coordinate system is chosen to eliminate the $K_p - K_q$ cross term. This choice results in the following expression for the tangent of the rotation angle:

$$\tan(2\vartheta) = \frac{2C_{xt}C_{yt}\ell_x\ell_y}{\ell_x^2(1 - C_{xt}^2) - \ell_y^2(1 - C_{yt}^2)} \ . \tag{65}$$

After a little algebra, it can be shown that the angle-Doppler GPSD in the $K_p - K_q$ coordinate system is

$$S_{KD}(K_p,K_q,\omega_D) = \sqrt{\pi}\,\tau_0 \exp\left[-\frac{\tau_0^2\omega_D^2}{4}\right] \tag{66}$$

$$\times \frac{\sqrt{\pi}\,\ell_p}{\sqrt{1 - C_{pt}^2}} \exp\left[-\frac{(K_p\ell_p - C_{pt}\tau_0\omega_D)^2}{4(1 - C_{pt}^2)}\right] \frac{\sqrt{\pi}\,\ell_q}{\sqrt{1 - C_{qt}^2}} \exp\left[-\frac{(K_q\ell_q - C_{qt}\tau_0\omega_D)^2}{4(1 - C_{qt}^2)}\right]$$

when the following definitions are used:

$$\frac{1}{\ell_p^2} = \frac{\cos^2\vartheta}{\ell_x^2} + \frac{\sin^2\vartheta}{\ell_y^2} \tag{67a}$$

$$\frac{1}{\ell_q^2} = \frac{\sin^2\vartheta}{\ell_x^2} + \frac{\cos^2\vartheta}{\ell_y^2} \tag{67b}$$

and

$$\frac{C_{pt}}{\ell_p} = \frac{\cos\vartheta\, C_{xt}}{\ell_x} + \frac{\sin\vartheta\, C_{yt}}{\ell_y} \tag{68a}$$

$$\frac{C_{qt}}{\ell_q} = -\frac{\sin\vartheta\, C_{xt}}{\ell_x} + \frac{\cos\vartheta\, C_{yt}}{\ell_y} \ . \tag{68b}$$

Two important features of the channel modeling technique developed for the general model are the evaluation of the signal power in this rotated coordinate system using simple error functions, and the use of the Doppler shifting property of the GPSD.

3.2.2 Computationally Efficient form of the Impulse Response Function.

Equation 66 contains terms of the form $K_p\,\ell_p - C_{pt}\tau_0\omega_D$ and $K_q\,\ell_q - C_{qt}\tau_0\omega_D$. Thus, except for the leading Gaussian term, the mean signal power at non-zero Doppler frequencies can be obtained from the GPSD evaluated at zero Doppler frequency and shifted in K_p–K_q space by the appropriate amount. This fact is used to reduce the computations necessary to generate realizations of the impulse response function.

To see the consequences of the shifting property, consider the impulse response function given by Equation 60, which is the basis of the channel simulation technique. In continuous notation and using the delta-function relationship between angle and delay, the random angle-Doppler-delay spectrum of the signal may be written as

$$\hat{h}(\boldsymbol{K}_\perp,\tau,\omega_D) = \begin{cases} \xi_N(\boldsymbol{K}_\perp,\omega_D)\sqrt{S_{KD}(\boldsymbol{K}_\perp,\omega_D)} & \text{if } \tau = \dfrac{\Lambda_y(K_x^2 + K_y^2)\ell_x^2}{4\omega_{coh}} \\ 0 & \text{otherwise} \end{cases} \tag{69}$$

where $\xi_N(\boldsymbol{K}_\perp,\omega_D)$ is white Gaussian noise with unity mean power and with the correlation properties

$$\langle \xi_N(\boldsymbol{K}_\perp,\omega_D)\xi_N^*(\boldsymbol{K}_\perp',\omega_D')\rangle = \delta(\boldsymbol{K}_\perp - \boldsymbol{K}_\perp')\,\delta(\omega_D - \omega_D') \tag{70a}$$

$$\langle \xi_N(\boldsymbol{K}_\perp,\omega_D)\xi_N(\boldsymbol{K}_\perp',\omega_D')\rangle = 0 \ . \tag{70b}$$

A key point is that the angle-Doppler GPSD, $S_{KD}(\boldsymbol{K}_\perp,\omega_D)$, may be written as

$$S_{KD}(\boldsymbol{K}_\perp,\omega_D) = S_D(\omega_D)\, S_{KC}\left[K_x - \frac{C_{xt}\tau_0\omega_D}{\ell_x},\ K_y - \frac{C_{yt}\tau_0\omega_D}{\ell_y}\right] \tag{71}$$

where

$$S_D(\omega_D) = \sqrt{\pi}\,\tau_0 \exp\left[-\frac{\tau_0^2\omega_D^2}{4}\right] \tag{72}$$

and

$$S_{KC}(\boldsymbol{K}_\perp) = \frac{\pi\,\ell_x\ell_y}{\sqrt{1 - C_{xt}^2 - C_{yt}^2}} \times \exp\left[-\frac{K_x^2\ell_x^2(1 - C_{yt}^2) + K_y^2\ell_y^2(1 - C_{xt}^2) + 2C_{xt}C_{yt}K_x\ell_xK_y\ell_y}{4(1 - C_{xt}^2 - C_{yt}^2)}\right] . \tag{73}$$

After inserting these expressions into Equation 60, the impulse response function is

$$h_A(\rho_0,\tau,t) = \int_{-\infty}^{\infty}\frac{dK_x}{2\pi}\int_{-\infty}^{\infty}\frac{dK_y}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega_D}{2\pi}\exp\left[i(K_xx_0 + K_yy_0 - \omega_Dt)\right] \tag{74}$$

$$\times\ g(K_x-K_{0x},K_y-K_{0y})\ \xi_N(K_x,K_y,\omega_D)$$

$$\times\ \left\{S_D(\omega_D)\, S_{KC}\left[K_x - \frac{C_{xt}\tau_0\omega_D}{\ell_x},\ K_y - \frac{C_{yt}\tau_0\omega_D}{\ell_y}\right]\right\}^{\frac{1}{2}}$$

where $\rho_0 = (x_0,y_0)$. Of course the delta-function relationship between angle and delay still holds although it is not explicitly shown in this equation.

The problem with this expression as written is that S_{KC} must be computed for each new Doppler frequency, which is time consuming. However S_{KC} can be evaluated once at zero Doppler frequency and shifted for non-zero frequencies. In a digital simulation this shifting is most efficiently done in discrete steps. Thus let

$$C_{xt}\tau_0\omega_D = m_x\Delta K_x\ell_x + \varepsilon_x\ell_x \tag{75a}$$

$$C_{yt}\tau_0\omega_D = m_y\Delta K_y\ell_y + \varepsilon_y\ell_y \tag{75b}$$

where ΔK_x and ΔK_y are the angular grid cell sizes that will be used to numerically evaluate Equation 74,

$$m_x = \text{int}\left[\frac{C_{xt}\tau_0\omega_D}{\ell_x \Delta K_x}\right] \tag{76a}$$

$$m_y = \text{int}\left[\frac{C_{yt}\tau_0\omega_D}{\ell_y \Delta K_y}\right] , \tag{76b}$$

and ε_x and ε_y are the residuals after the discrete shifts. The function int $(\cdot)$ is equal to the integer part oi the argument. Now define a shifted angle-Doppler GPSD, denoted S_{KS}:

$$S_{KS}(\boldsymbol{K}_\perp) = S_{KC}\left[K_x - \frac{C_{xt}\tau_0\omega_D}{\ell_x}, K_y - \frac{C_{yt}\tau_0\omega_D}{\ell_y}\right] . \tag{77}$$

After substituting this into the equation for the impulse response function and changing angular variables to

$$K_x' = K_x - \varepsilon_x \tag{78a}$$

$$K_y' = K_y - \varepsilon_y , \tag{78b}$$

Equation 74 becomes

$$\begin{aligned} h_A(\rho_0,\tau,t) = & \int_{-\infty}^{\infty} \frac{dK_x'}{2\pi} \int_{-\infty}^{\infty} \frac{dK_y'}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_D}{2\pi} \exp\left\{ i[(K_x' + \varepsilon_x)x_0 + (K_y' + \varepsilon_y)y_0 - \omega_D t]\right\} \\ & \times g(K_x' + \varepsilon_x - K_{0x}, K_y' + \varepsilon_y - K_{0y})\ \xi_N(K_x' + \varepsilon_x, K_y' + \varepsilon_y, \omega_D) \\ & \times \sqrt{S_D(\omega_D)\ S_{KS}(K_x', K_y')} \ . \end{aligned} \tag{79}$$

We have ignored the residual shift in the arguments of S_{KS} so this function is the result of a discrete shift of the function S_{KC}. Equation 79, in discrete form, is used to generate the impulse response functions at the outputs of multiple antennas.

3.2.3 Discrete Evaluation of the GPSD.

The first step in generating the impulse response function at the output of an antenna is the evaluation of the GPSD on a discrete $K_x - K_y - \omega_D$ grid. The delta-function relationship between angle and delay is used to relate signal components within an angular annulus to a particular delay bin. Thus at this point it is not necessary to include delay explicitly, and the GPSD can be integrated over this variable. To assure conservation of energy (or, more correctly, to conserve signal power), the GPSD is integrated over each $K_x - K_y - \omega_D$ grid cell, and that power is assigned to the center point of the cell. A procedure for efficiently performing the three-dimensional integral is described in this subsection.

The two-dimensional angle and Doppler frequency grids are defined by the equations

$$K_x = k_x \Delta K_x \qquad (-N_x/2 \le k_x \le N_x/2-1) \tag{80a}$$

$$K_y = k_y \Delta K_y \qquad (-N_y/2 \le k_y \le N_y/2-1) \tag{80b}$$

$$\omega_D = k_D \Delta \omega_D \qquad (-N_D/2 \le k_D \le N_D/2-1) \ . \tag{80c}$$

The requirements on the grid cell sizes ΔK_x, ΔK_y, and $\Delta \omega_D$ and on the number of grid cells N_x, N_y, and N_D will be discussed later.

The mean signal power in each angle-Doppler frequency grid cell at the output of an antenna is then given by the integral of the antenna filtered power spectral density function over each grid cell:

$$E_A(k_x,k_y,k_D) = \int_{(k_x-1/2)\Delta K_x}^{(k_x+1/2)\Delta K_x} \frac{dK_x}{2\pi} \int_{(k_y-1/2)\Delta K_y}^{(k_y+1/2)\Delta K_y} \frac{dK_y}{2\pi} \int_{(k_D-1/2)\Delta \omega_D}^{(k_D+1/2)\Delta \omega_D} \frac{d\omega_D}{2\pi} S_A(\boldsymbol{K}_\perp,\tau,\omega_D) \ . \tag{81}$$

Equation 81 is completely general, but it requires that the triple integral be computed and stored separately for each antenna with different beamwidths or pointing angles. If it is assumed that the antenna beam pattern is constant over a $K_x - K_y - \omega_D$ grid cell, then this equation can be approximated by

$$E_A(k_x,k_y,k_D) = G(k_x \Delta K_x - K_{0x},\ k_y \Delta K_y - K_{0y})\, E_{KD}(k_x,k_y,k_D) \tag{82}$$

where $E_{KD}(k_x,k_y,k_D)$ is the incident signal power in the $K_x - K_y - \omega_D$ grid cell. The accuracy of this approximation is addressed in Appendix A where it is shown to conserve energy (or power) to within a small fraction of a percent.

To produce an efficient channel model, the quantity $E_{KD}(k_x,k_y,k_D)$ must be readily evaluated. This is a major problem of the general model. Straightforward evaluation of E_{KD} requires a closed-form expression for the triple integral

$$E_{KD}(k_x,k_y,k_D) = \int_{(k_x-1/2)\Delta K_x}^{(k_x+1/2)\Delta K_x} \frac{dK_x}{2\pi} \int_{(k_y-1/2)\Delta K_y}^{(k_y+1/2)\Delta K_y} \frac{dK_y}{2\pi} \int_{(k_D-1/2)\Delta \omega_D}^{(k_D+1/2)\Delta \omega_D} \frac{d\omega_D}{2\pi} S_{KD}(\boldsymbol{K}_\perp,\omega_D) \tag{83}$$

where the integrand is given by Equations 71-73. Clearly the $K_x - K_y$ and angle-Doppler cross terms in the expression for $S_{KD}(\boldsymbol{K}_\perp,\omega_D)$ do not allow a simple closed form expression for E_{KD} for the general case, although such expressions can be obtained in the frozen-in and turbulent limits.

Two "tricks" are used to evaluate Equation 83 efficiently. The first trick is to take advantage of the translational properties of the GPSD described in the previous subsection. The power in a $K_x - K_y - \omega_D$ grid cell is

$$E_{KD}(k_x,k_y,k_D) = \int_{(k_D - 1/2)\Delta\omega_D}^{(k_D + 1/2)\Delta\omega_D} \frac{d\,\omega_D}{2\pi} S_D(\omega_D) \tag{84}$$

$$\times \int_{(k_x - 1/2)\Delta K_x}^{(k_x + 1/2)\Delta K_x} \frac{dK_x}{2\pi} \int_{(k_y - 1/2)\Delta K_y}^{(k_y + 1/2)\Delta K_y} \frac{dK_y}{2\pi} S_{KC}\left[K_x - \frac{C_{xt}\tau_0\omega_D}{\ell_x}, K_y - \frac{C_{yt}\tau_0\omega_D}{\ell_y}\right].$$

The key to simplifying this expression is to note that the Doppler frequency grid cell size is relatively small because of the large number of Doppler samples required to produce a long time realization. Thus it can be assumed that S_{KC} is constant over a Doppler frequency cell, and Equation 84 reduces to

$$E_{KD}(k_x,k_y,k_D) = E_D(k_D)\, E_{KC}(k_x - m_x,\, k_y - m_y) \tag{85}$$

where $E_D(k_D)$ in terms of complimentary error functions, erfc $(\cdot)$, is

$$E_D(k_D) = \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{(k_D - 1/2)\tau_0\Delta\omega_D}{2}\right] - \mathrm{erfc}\left[\frac{(k_D + 1/2)\tau_0\Delta\omega_D}{2}\right]\right\}. \tag{86}$$

The quantity $E_{KC}(k_x - m_x, k_y - m_y)$ is the power in a shifted $K_x - K_y$ grid cell. Because of the $K_x - K_y$ cross terms in the expression for S_{KC}, an easily evaluated closed-form result still has not been obtained for E_{KC}.

The second trick used in the channel modeling technique is to note that a rotation by the angle ϑ (Eqn. 65) in the $K_x - K_y$ plane produces an orthogonal form of the GPSD that does not contain angular cross terms, and is therefore readily integrated. This orthogonalized GPSD is given by Equation 66, which has the following form for its angular part:

$$S_{KC}(K_p,K_q) = \frac{\pi\,\ell_p\,\ell_q}{\sqrt{(1 - C_{pt}^2)(1 - C_{qt}^2)}} \exp\left[-\frac{K_p^2\ell_p^2}{4(1 - C_{pt}^2)} - \frac{K_q^2\ell_q^2}{4(1 - C_{qt}^2)}\right]. \tag{87}$$

The signal power in a $K_p - K_q$ grid cell, with indices k_p and k_q respectively, is

$$E_{KC}(k_p,k_q) = E_p(k_p)\, E_q(k_q) \tag{88}$$

where

$$E_p(k_p) = \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{(k_p - 1/2)\Delta K_p \ell_p}{2\sqrt{1 - C_{pt}^2}}\right] - \mathrm{erfc}\left[\frac{(k_p + 1/2)\Delta K_p \ell_p}{2\sqrt{1 - C_{pt}^2}}\right]\right\} . \tag{89}$$

A similar expression holds for $E_q(k_q)$.

Now $E_{KC}(k_p,k_q)$ can be computed on a fine $K_p - K_q$ grid, and the values simply assigned to the $K_x - K_y$ grid cell in which they fall. The $K_x - K_y$ cell indices are computed as follows:

$$k_x = \mathrm{int}\left[\frac{k_p \Delta K_p \cos\vartheta - k_q \Delta K_q \sin\vartheta}{\Delta K_x}\right] \tag{90a}$$

$$k_y = \mathrm{int}\left[\frac{k_p \Delta K_p \sin\vartheta + k_q \Delta K_q \cos\vartheta}{\Delta K_y}\right] . \tag{90b}$$

The total power in a $K_x - K_y$ grid cell is then the sum of all $E_{KC}(k_p,k_q)$ values that fall within the $K_x - K_y$ cell. Roughly ten $K_p - K_q$ grid cells are required within each $K_x - K_y$ cell for this brute-force procedure to work. Thus the $K_p - K_q$ cell sizes are determined by the expressions

$$\Delta K_p = \frac{0.1}{\left[\dfrac{\cos^2\vartheta}{(\Delta K_x)^2} + \dfrac{\sin^2\vartheta}{(\Delta K_y)^2}\right]^{\frac{1}{2}}} \tag{91a}$$

$$\Delta K_q = \frac{0.1}{\left[\dfrac{\sin^2\vartheta}{(\Delta K_x)^2} + \dfrac{\cos^2\vartheta}{(\Delta K_y)^2}\right]^{\frac{1}{2}}} . \tag{91b}$$

A detailed description of the algorithms used to compute $E_{KC}(k_x - m_x, k_y - m_y)$ and to shift this array for different Doppler frequencies is given in Appendix B.

3.2.4 Random Realizations.

The next step in the channel modeling technique is to generate a random realization of the angle-Doppler spectrum of the impulse response function and to assign the spectral components to delay bins using the delta-function relationship between angle and delay. Discrete Fourier transforms are then performed to obtain the impulse response function.

Assignment of Angular Spectral Components to Delay Bins. The delta-function relationship between angle and delay (Eqn. 62) in the diffraction-limited form of the GPSD is used to assign angular spectral components to discrete delay bins. This form of the GPSD is non-zero only when

$$\tau = \frac{\Lambda_y (K_x^2 + K_y^2) \ell_x^2}{4\omega_{coh}} \; . \tag{92}$$

A straightforward approach to assigning angular spectral components to delay bins is to compute the right-hand side of Equation 92 at the center of each $K_x - K_y$ grid cell and to compute the index of the delay bin using

$$j = \mathrm{int}\left[\frac{\tau}{\Delta\tau}\right] = \mathrm{int}\left[\frac{\Lambda_y (K_x^2 + K_y^2) \ell_x^2}{4\omega_{coh}\Delta\tau}\right] \tag{93}$$

where $\Delta\tau$ is the sample size of the delay bins. The problem with this approach is that when the delay sample size is sufficiently small, the number of angular spectral components that fall within a delay bin may vary substantially from one delay bin to the next producing ragged statistics. A simple solution to this problem is to randomly wiggle the angular grid cell centers before applying Equation 93. This spreads angular spectral components more or less uniformly into any one of several delay bins, and results in better agreement between the ensemble signal power in a delay bin and the realization power in that bin. The randomly-wiggled angular grid cell centers are computed as

$$K_x = \left[k_x + \xi_{Ux} - \frac{1}{2}\right] \Delta K_x \tag{94a}$$

$$K_y = \left[k_y + \xi_{Uy} - \frac{1}{2}\right] \Delta K_y \tag{94b}$$

where ξ_{Ux} and ξ_{Uy} are independent, uniformly distributed random numbers on the interval $[0,1)$. These wiggled cell centers are then used in Equation 93 to compute the corresponding j index of the delay grid.

In this way each angular spectral component is assigned to a unique delay bin. Because the angular spectral components are uncorrelated, this procedure also guarantees that the impulse response function is uncorrelated from one delay bin to another.

Discrete Representation of Impulse Response Function. The discrete impulse response function is defined on a discrete time and delay grids defined by the equations

$$t = k_T \Delta t \qquad (k_T = 1, 2, \cdots, N_T) \tag{95}$$

$$\tau = j\Delta\tau \; . \qquad (j = 0, 1, \cdots, N_\tau - 1) \tag{96}$$

The requirements on the discrete time step Δt, the number to time steps N_T, the delay sample size $\Delta\tau$, and the number of delay bins N_τ will be discussed in Section 3.3.

Equation 79 gives the impulse response function in terms of Fourier transforms from the random angle-Doppler spectral components to the time domain at the location of the phase center of each antenna. In discrete form, this equation is

$$h_A(j\Delta\tau, k_T\Delta t) = \frac{8\pi^3}{\Delta K_x \Delta K_y \Delta\omega_D \Delta\tau} \sum_{k_D = -N_D/2}^{N_D/2-1} \frac{\Delta\omega_D}{2\pi} \sum_{k_x = -N_x/2}^{N_x/2-1} \frac{\Delta K_x}{2\pi} \sum_{k_y = -N_y/2}^{N_y/2-1} \frac{\Delta K_y}{2\pi}$$

$$\times \ \exp\left\{ i[(k_x\Delta K_x + \varepsilon_x)x_0 + (k_y\Delta K_y + \varepsilon_y)y_0 - k_D\Delta\omega_D k_T\Delta t] \right\}$$

$$\times \ g(k_x\Delta K_x + \varepsilon_x - K_{0x}, k_y\Delta K_y + \varepsilon_y - K_{0y})\ \xi_N(k_x, k_y, k_D)$$

$$\times \ \sqrt{E_D(k_D)\ E_{KS}(k_x\Delta K_x, k_y\Delta K_y)} \ . \tag{97}$$

The normalization factor, $8\pi^3/\Delta K_x\Delta K_y\Delta\omega_D\Delta\tau$, has been chosen so that $h_A(j\Delta\tau,k_T\Delta t)\Delta\tau$ represents the received signal during the delay interval $j\Delta\tau$ to $(j+1)\Delta\tau$. As the K_x and K_y sums are performed, Equation 93 is used to assign angular spectral components to delay bins. Then the signal components in each delay bin are Fourier transformed from the Doppler domain to the time domain.

The quantity $\xi_N(k_x,k_y,k_D)$ is a complex, zero-mean, Gaussian random variable with the properties

$$\langle \xi_N(a,b,c)\xi_N^*(\alpha,\beta,\gamma)\rangle = \delta_{a,\alpha}\,\delta_{b,\beta}\,\delta_{c,\gamma} \tag{98a}$$

$$\langle \xi_N(a,b,c)\rangle = 0 \tag{98b}$$

$$\langle \xi_N(a,b,c)\xi_N(\alpha,\beta,\gamma)\rangle = 0 \tag{98c}$$

where $\delta_{m,n}$ is the Kronecker delta symbol. A convenient method of generating the complex, zero-mean, Gaussian random numbers is to use the following equation

$$\xi_N = \sqrt{-P_0 \ln(\xi_{U1})}\, \exp\,(2\pi i \xi_{U2}) \tag{99}$$

where ξ_{U1} and ξ_{U2} are independent random numbers uniformly distributed on the interval $[0,1)$, and P_0 is the mean power of the random samples ($P_0 = \langle \xi_N \xi_N^* \rangle = 1$ for this application).

In comparing the discrete equation for the impulse response function with its continuous-variable analog (Eqn. 79), note that the residual shifts ε_x and ε_y may be ignored in the arguments of the random spectral components, ξ_N, because shifted white Gaussian noise is still white Gaussian noise.

Elimination of the DC Component. As written, Equation (97) for the discrete impulse response function allows a Doppler spectral component with zero-Doppler

frequency (i.e., the $k_D = 0$ component). This component will in turn result in a DC component in the time domain realization, which may be undesirable, particularly if the DC component is large.

A simple solution to this problem is to set the zero-Doppler frequency component of $\xi_N(k_x,k_y,k_D)$ to zero. Just doing this, however, results in a reduction in the mean power in the impulse response function by the zero-Doppler frequency power, $E_D(0)$.

This latter problem can be solved simply by allowing the Doppler frequency bins adjacent to zero Doppler to expand in size. Thus the first positive Doppler frequency bin encompasses frequencies 0 to $3\Delta\omega_D/2$ and has power $E_D(1)+E_D(0)/2$. Similarly, the first negative Doppler frequency bin encompasses frequencies $-3\Delta\omega_D/2$ to 0 and has power $E_D(-1)+E_D(0)/2$. All other Doppler frequency bins encompass frequencies $(k_Dk_D-1/2)\Delta\omega_D$ to $(k_D+1/2)\Delta\omega_D$ and have power $E_D(k_D)$.

3.3 GRIDS.

Angle and Doppler frequency grid sizes are determined by requiring that the angle-Doppler frequency grid encompass a large fraction, say 0.999, of the power in the GPSD of the signal. The 0.001 error must then be allocated between the angular and Doppler parts of the three-dimensional grid. An arbitrary, but intuitively reasonable, allocation is to divide the error equally between the angular and Doppler frequency parts of the grid and equally between the two angular components. Thus the Doppler frequency grid limits are determined by requiring that the Doppler frequency grid encompass $(0.999)^{1/2}$ of the Doppler frequency power, and each angular grid must encompass $(0.999)^{1/4}$ of the angular power.

The angular and Doppler frequency power spectra are all Gaussian. Thus each can separately be written in the form

$$S(\kappa) = \sqrt{\pi}\exp\left[-\frac{\kappa^2}{4}\right] \tag{100}$$

where κ is a normalized angle or Doppler frequency (i.e., κ is equal to $K_x\ell_x$, $K_y\ell_y$ or $\tau_0\omega_D$). In order for a κ grid to encompass a fraction ζ_0 of the signal power, it must extend from $-\kappa_{max}$ to $+\kappa_{max}$ where

$$\zeta_0 = \int_{-\kappa_{max}}^{\kappa_{max}} S(\kappa)\, d\kappa \ . \tag{101}$$

This equation is easily solved for κ_{max} in terms of ζ_0 with the result

$$\kappa_{max} = 2\,\mathrm{erf}^{-1}(\zeta_0) \tag{102}$$

where $\mathrm{erf}^{-1}(\cdot)$ is the inverse error function. If ζ_0 is chosen to be $(0.999)^{1/2}$ for the Doppler frequency grid, then

$$\kappa_{D,max} = 2 \times 2.4612 = 4.9224 \ , \tag{103a}$$

and if ζ_0 is chosen to be $(0.999)^{1/4}$ for either angular grid, then

$$\kappa_{K,max} = 2 \times 2.5895 = 5.1790 \ . \tag{103b}$$

A second requirement on the sizes of the angular, Doppler frequency, and delay grids is that they be defined at the output of the antennas, thereby eliminating regions of the grids that contribute to the signal power incident on the antennas but, because of antenna filtering, do not contribute to the power of the output signals. Much of the complexity of the algorithms used to determine grid sizes is a result of this requirement, but computing grid sizes in this way results in a substantial reduction in the size of the grids, and therefore in computation time, when antenna filtering effects are large.

The numbers of cells in the angular (N_x and N_y), delay (N_τ), and time (N_T) grids are inputs to the channel simulation. The delay sample size ($\Delta\tau$) and the number of samples per decorrelation time (N_0) are also inputs. From these quantities and the channel and antenna parameters, the angular (ΔK_x and ΔK_y), Doppler frequency ($\Delta\omega_D$), and time (Δt) grid cell sizes and the required number of Doppler frequency samples (N_D) are computed. Requirements on input grid parameters and consistency checks on computed grid parameters are described in the next subsections.

3.3.1 Angular Grid.

Angular grid sizes are determined by the requirement that the fraction $(0.999)^{1/2}$ (i.e., 0.9995) of the signal power after antenna filtering be contained in the two-dimensional angular grid. First consider the K_x grid. Because of the symmetry in the angular grid, the requirements on the K_y grid can then be obtained by analogy.

The K_x power spectrum at the output of an antenna is

$$S_A(K_x) = \int_{-\infty}^{\infty} \frac{dK_y}{2\pi} \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} \frac{d\omega_D}{2\pi} G(K_x - K_{0x}, K_y - K_{0y})\, S(\boldsymbol{K}_\perp, \tau, \omega_D) \tag{104}$$

$$= \frac{\sqrt{\pi}\,\ell_x}{\sqrt{Q_y}} \exp\left[- \alpha_u^2 K_{0u}^2 - \alpha_v^2 K_{0v}^2 - \frac{Q_0^2}{Q_y} \frac{K_x^2 \ell_x^2}{4} + \left(A_x - \frac{A_y Q_{xy}}{Q_y} \right) \frac{K_x \ell_x}{2} + \frac{A_y^2}{4 Q_y} \right]$$

where

$$A_x = (Q_x - 1)K_{0x}\ell_x + Q_{xy}K_{0y}\ell_y \tag{105a}$$

$$A_y = Q_{xy}K_{0x}\ell_x + (Q_y - 1)K_{0y}\ell_y \ . \tag{105b}$$

The limits of the K_x grid are determined by requiring that

$$\zeta_0 P_A = \int_{-K_{x,max}}^{K_{x,max}} S_A(K_x)\,\frac{dK_x}{2\pi} \ . \tag{106}$$

If the coefficient of the linear K_x term in the exponent of Equation 104 is positive then the lower limit of the integral in Equation 106 can be replaced by $-\infty$. Conversely, if the coefficient is negative then the upper limit can be replaced by $+\infty$. Either way, the result for ζ_0 is

$$\zeta_0 = \frac{1}{2}\left\{1 + \mathrm{erf}\left[\frac{Q_0 K_{x,max}\ell_x}{2\sqrt{Q_y}} - \frac{|A_x Q_y - A_y Q_{xy}|}{2Q_0\sqrt{Q_y}}\right]\right\} \tag{107}$$

where erf $(\cdot)$ is the error function. Setting ζ_0 equal to $(0.999)^{1/4}$ and solving for $K_{x,max}$ gives the following approximate result:

$$K_{x,max} = \frac{\kappa_{K,max}}{\ell_{Ax}} + \frac{|A_x Q_y - A_y Q_{xy}|}{\ell_{Ax} Q_0 \sqrt{Q_y}} \ . \tag{108a}$$

Similarly, the limit of the K_y grid is

$$K_{y,max} = \frac{\kappa_{K,max}}{\ell_{Ay}} + \frac{|A_y Q_x - A_x Q_{xy}|}{\ell_{Ay} Q_0 \sqrt{Q_x}} \ . \tag{108b}$$

The first terms in the expressions for $K_{x,max}$ and $K_{y,max}$ give the required extent of the grid when the antenna is pointed along the line-of-sight. The second terms, which are non-zero only when the antenna is pointed away from the line-of-sight, give the amount by which the grid must be extended in order for the grid to encompass the beam.

Clearly $K_{x,max}$ and $K_{y,max}$ depend on antenna beamwidths and pointing angles because of the A and Q factors. If there are multiple antennas, $K_{x,max}$ and $K_{y,max}$ must be computed for each antenna. The largest values are then used to determine the boundaries of the angular grid.

The angular grid cell sizes can now be computed as

$$\Delta K_x = \frac{2K_{x,max}}{N_x} \tag{109a}$$

$$\Delta K_y = \frac{2K_{y,max}}{N_y} \tag{109b}$$

where the number of grid cells, N_x and N_y, are inputs to the channel model.

A reasonable minimum value for the number of K_x or K_y grid cells is 32. However, this number may not be sufficient if there are multiple antennas with different phase center locations. Consider the two antennas with the largest separations, d_x and d_y, in the x–y plane. Because the impulse response functions at the outputs of the two antennas are generated by discrete Fourier transforms from the angular domain to the phase center locations of the antenna, the unambiguous distances, $2\pi/\Delta K_x$ and $2\pi/\Delta K_y$, of the DFTs must exceed the maximum antenna separations, say by a factor of 2. This requirement puts upper limits on ΔK_x and ΔK_y:

$$\Delta K_x \le \frac{\pi}{d_x} \tag{110a}$$

$$\Delta K_y \le \frac{\pi}{d_y} \ . \tag{110b}$$

If these criteria are not met, then N_x and/or N_y must be increased, thereby decreasing ΔK_x and/or ΔK_y, until they are met. The minimum required values for N_x and N_y can then be written as

$$N_x = \max\left[32, \frac{2d_x K_{x,max}}{\pi}\right] \tag{111a}$$

$$N_y = \max\left[32, \frac{2d_y K_{y,max}}{\pi}\right] \ . \tag{111b}$$

3.3.2 Doppler Frequency and Time Grids.

The Doppler frequency grid size is also determined by the requirement that 99.9 percent of the signal power after antenna filtering be contained in the angle-Doppler frequency grid. Thus the antenna-filtered Doppler power spectrum is required. This function is most easily obtained by noting that the temporal coherence function has the form

$$\Gamma_A(t) = P_A \exp\left[-\frac{t^2}{\tau_A^2} + i\omega_A t\right] \ . \tag{112}$$

Recall from Section 2.3 that ω_A is the mean Doppler shift due to antenna pointing. The Doppler power spectrum is then given by the equation

$$S_A(\omega_D) = \int_{-\infty}^{\infty} \Gamma_A(t)\, dt = \sqrt{\pi}\, \tau_A\, P_A \exp\left[- \frac{\tau_A^2\, (\omega_D + \omega_A)^2}{4} \right] . \tag{113}$$

The calculation of the limits on the Doppler frequency grid is exactly analogous to that done for the angular grid. The limits of the ω_D grid are determined by setting ζ_0 equal $(0.999)^{1/2}$ in the equation

$$\zeta_0 P_A = \int_{-\omega_{D,max}}^{\omega_{D,max}} S_A(\omega_D)\, \frac{d\omega_D}{2\pi} . \tag{114}$$

Solving for ζ_0,

$$\zeta_0 = \frac{1}{2} \left\{ 1 + \operatorname{erf} \left[\frac{\tau_A(\omega_{D,max} - |\omega_A|)}{2} \right] \right\} , \tag{115}$$

gives the following approximate result for $\omega_{D,max}$:

$$\omega_{D,max} = \frac{\kappa_{D,max}}{\tau_A} + |\omega_A| . \tag{116}$$

The first term in this expression gives the required maximum Doppler frequency when the antenna is pointed along the line-of-sight, and the second term is the result of the mean Doppler shift effect of antenna pointing. If there are multiple antennas, $\omega_{D,max}$ must be calculated for each antenna, and the largest value used to determine the Doppler frequency grid size.

At this point the required number of Doppler frequencies, N_D, is still unspecified. However, because a fast Fourier transform (FFT) will be used to transform from Doppler frequency to time, the time grid requirements may be used to derive the Doppler frequency grid cell size $\Delta\omega_D$ and then the required number of Doppler frequency samples.

Consider the requirements on the time samples. *Dana* [1982, 1988] has shown that at least 10 samples per decorrelation time are required to reproduce accurately the temporal statistics of Rayleigh fading. This is also a DNA requirement on Rayleigh fading realizations of the impulse response function [*Wittwer* 1980]. The time grid cell size is then

$$\Delta t = \frac{\tau_{A,min}}{N_0} \tag{117}$$

where N_0 is the number of samples per decorrelation time (N_0 must be greater than or equal to 10), and $\tau_{A,min}$ is the smallest value of the filtered decorrelation time for all antennas.

In addition, DNA requires that there be at least 100 decorrelation times in all realizations of the impulse response function at the antenna output. Thus

$$N_T \geq \frac{100\,\tau_{A,max}}{\Delta t} \tag{118}$$

where $\tau_{A,max}$ is the largest value of the filtered decorrelation time for all antennas. If this condition is not met, the number of time samples, N_T, must be increased. It is also necessary that N_T be equal to a power of 2 to use an FFT. The minimum value of N_T is then 1024 to meet the requirement of Equation 118 with N_0 equal to 10. Further discussion of the required number of time samples is in Appendix C.

Because of the FFT relationship between the time and Doppler frequency domains, the Doppler frequency grid cell size is

$$\Delta\omega_D = \frac{2\pi}{N_T \Delta t} \ . \tag{119}$$

The minimum number of Doppler frequency samples necessary for the grid to encompass the maximum required Doppler frequency is then given by

$$N_D = \frac{2\,\omega_{D,max}}{\Delta\omega_D} = \frac{N_T\,\omega_{D,max}\,\tau_{A,min}}{\pi N_0} \ . \tag{120}$$

In general, N_D will be smaller than N_T implying that fewer than N_T Doppler frequency samples are required. The Doppler frequency arrays may then be zero-padded to N_T samples before the FFT is performed. If, however, N_D is greater than N_T, the implication is that there are too few samples per decorrelation time, and N_0 must be increased.

The minimum number of Doppler frequency samples can be computed from Equations 116 and 120. If all antennas are pointed along the line-of-sight, then the minimum value of N_D is

$$N_{D,min} = \frac{\kappa_{D,max} N_T}{\pi N_0} \ . \tag{121}$$

For realizations with 100 decorrelation times (N_T/N_0 = 100), the minimum number of Doppler frequency samples is approximately 150. If antennas are pointed away from the line-of-sight, the required number of Doppler frequency samples will increase beyond 150.

3.3.3 Delay Grid.

The delay sample size is usually chosen on the basis of the modulation bandwidth of the transmitted signal, and is therefore not a parameter that is under the direct control of the channel simulation. For example, in a phase-shift keying (PSK) application there must be at least two delay samples per channel symbol to simulate accurately the transmitted frequency spectrum. The delay sample size is then generally chosen to be equal to one-half the channel symbol period. In a frequency-shift keying (FSK) application with frequency hopping, the delay sample size is chosen so that the unambiguous frequency bandwidth of the impulse response function, $1/\Delta\tau$, exceeds the hopping bandwidth by some comfortable margin.

The number of delay bins is an input to the channel simulation. The requirement on both N_τ and $\Delta\tau$ is that the realization delay grid size $N_\tau\Delta\tau$ encompass at least 97.5 percent of the delayed signal power at the outputs of the antennas. A somewhat smaller percentage is used to define the limits of the delay grid than is used to define other grid limits because of the slower decay of signal power with delay than with angle and Doppler frequency (exponential decay with delay versus Gaussian decay with angle and Doppler frequency).

The ensemble signal power in the delay bins is given by the integral

$$P_j = \int_{j\Delta\tau}^{(j+1)\Delta\tau} S_A(\tau)\, d\tau \tag{122}$$

where $S_A(\tau)$ is the delay power spectral density at the output of an antenna. The general expression for $S_A(\tau)$ is quite complicated, especially when the antenna is pointed away from the line-of-sight, and will not be given here. The reader is referred to *Frasier* [1990] for details on $S_A(\tau)$.

The total signal power in the delay grid,

$$P_\tau = \sum_{j=0}^{N_\tau - 1} P_j \ , \tag{123}$$

must be greater than or equal to $0.975P_A$. If not, either N_τ or $\Delta\tau$ or both must be increased.

An *estimate* of the required maximum realization delay grid size, $N_\tau\Delta\tau$, can be obtained by considering the simple case of isotropic scattering and isotropic antennas pointed along the line-of-sight. In this case, the delay power spectral density is

$$S_A(\tau) = \omega_{coh} \exp(-Q\omega_{coh}\tau) \tag{124}$$

where Q is given by Equation (51). The maximum required delay is computed by inverting the equation:

$$\int_{0}^{\tau_{max}} S_A(\tau)\, d\tau = \zeta_0 P_A \ , \tag{125}$$

where ζ_0 is equal to 0.975.

An estimate of the maximum required delay is:

$$\tau_{max} = N_\tau \Delta\tau = \frac{-\ln(1-\zeta_0)}{2\pi f_0 Q} = \frac{3.7}{2\pi f_A} \ . \tag{126}$$

This equation is valid only for the conditions listed above. However, ACIRF will compute the required maximum delay for the actual input antenna configurations and channel parameters, and will compare its computed value of τ_{max} with the input value of maximum delay, $N_\tau \Delta\tau$. A warning message will advise the user if $N_\tau \Delta\tau$ is too big, and the code will stop if $N_\tau \Delta\tau$ is too small.

SECTION 4
MATCHED FILTER EXAMPLES

Examples of the received voltage out of a filter matched to a transmitted square pulse are presented in this section. These examples are intended to illustrate the effects of frequency selectivity and antenna filtering on transionospheric communication links and to illustrate the differences in the structure of the received signal depending on whether the frozen-in, turbulent, or general models are used to generate the impulse response function realizations. The following subsection also illustrates how the received voltage can be constructed from the impulse response function realizations in a digital link simulation. Additional examples for specific system applications may be found in *Bogusch, et. al.* [1981] and *Bogusch, Guigliano, and Knepp* [1983].

4.1 MATCHED FILTER OUTPUT SIGNAL.

The output of a matched filter can be constructed by convolving the impulse response function of the channel and antenna with the combined impulse response function of the transmitter and receiver. A second approach is to construct the combined frequency response of the transmitter, channel, antenna, and receiver and then to Fourier transform that result to obtain the matched-filter output. This latter approach is used for the examples presented in this section.

The starting point is to calculate the channel/antenna transfer function that is the Fourier transform of the impulse response function:

$$H(\omega,t) = \int_0^\infty h(\tau,t) \exp(-i\omega\tau)\, d\tau \ . \tag{127}$$

This function represents the response of the channel and antenna at time t to a transmitted sinewave with radian frequency ω.

For a transmitted square pulse with a chip duration T_C, the received voltage out of the matched filter at time t can be written as

$$r(\tau,t) = \int_{-\infty}^{\infty} M(\omega)\, H(\omega,t) \exp(i\omega\tau) \frac{d\omega}{2\pi} \tag{128}$$

where τ is the time delay of the matched filter relative to the nominal time-of-arrival (i.e., the time-of-arrival under benign propagation conditions). The combined spectrum of the transmitted square pulse and the receiver matched filter is

$$M(\omega) = T_C \frac{\sin^2(\omega T_C/2)}{(\omega T_C/2)^2} \ . \tag{129}$$

The impulse response function is generated with N_τ delay samples of size $\Delta\tau$, so the discrete channel/antenna transfer function has an unambiguous frequency response of $2\pi/\Delta\tau$ radians. If this bandwidth is divided into N_F frequency samples, the discrete channel/antenna transfer function at time $k_T\Delta t$ is

$$H(k_F\Delta\omega,k_T\Delta t) = \sum_{j=0}^{N_\tau-1} h(j\Delta\tau,k_T\Delta t)\Delta\tau \exp\left[-ij\Delta\tau k_F\Delta\omega\right] \tag{130}$$

where $\Delta\omega = 2\pi/(N_F\Delta\tau)$. Recall that the normalization of the impulse response function is such that the factor $\Delta\tau$ following $h(j\Delta\tau,k_T\Delta t)$ must be included. The range of the index k_F in this equation is from $-N_F/2$ to $N_F/2-1$ representing a range of frequencies from $-N_F\Delta\omega/2$ to $(N_F-1)\Delta\omega/2$. Of course the number of frequency samples should be greater than or equal to the number of delay samples in order for the transfer function to preserve the information contained in the impulse response function. If the minimum required number of delay samples is chosen, it may be necessary to select the number of frequency samples to be greater than the number of delay samples to minimize delay aliasing of the matched-filter output.

The voltage out of the matched filter, versus time and relative delay, is given by:

$$r(\tau,k_T\Delta t) = \frac{\Delta\omega T_C}{2\pi}\sum_{k_F=-N_F/2}^{N_F/2-1}\left[\frac{\sin^2(k_F\Delta\omega T_C/2)}{(k_F\Delta\omega T_C/2)^2}\right]H(k_F\Delta\omega,k_T\Delta t)\exp(ik_F\Delta\omega\tau) \ . \tag{131}$$

If the delay sample size $\Delta\tau$ of the realization of the impulse response function is chosen to be $T_C/2$, then $\Delta\omega T_C/2 = 2\pi/N_F$ and $r(\tau,k_T\Delta t)$ represents a signal that is band-limited to the frequency range $-1/T_C$ to $+1/T_C$. Note that the matched-filter output $r(\tau,k_T\Delta t)$ is unambiguous in delay over the interval from 0 to $(N_F-1)\Delta\tau$ compared to the delay interval of 0 to $(N_\tau-1)\Delta\tau$ for the original realization.

Delay in Equation 131 is a continuous variable, and any value may be used. In a digital simulation of a receiver, this value could be determined by the output of a delay-lock tracking loop. In the matched-filter output amplitude plots below, delay is varied over the entire unambiguous range to produce an array of $r(\tau,k_T\Delta t)$ values.

In the examples that follow, the chip rate R_C ($R_C = 1/T_C$) is set at 1 MHz, and the random realizations of the impulse response function are generated with a delay sample size of $T_C/2$. However, the frequency selective effects depend only on the ratio of the frequency selective bandwidth to the chip rate f_0/R_C. For antenna examples, a uniformly-weighted circular antenna and isotropic scattering are assumed. Antenna effects then depend only on the ratio of the antenna diameter D to the decorrelation distance ℓ_0 and the antenna pointing angle.

4.2 FREQUENCY SELECTIVE EFFECTS.

In a high data rate communications link, the major effect of frequency selective fading is intersymbol interference. Even relatively small amounts of delay spread can catastrophically degrade demodulation performance in such a link using conventional matched-filter detection techniques.

Figure 11 shows examples of the matched-filter output amplitude for three levels of frequency selective propagation disturbances, characterized by the ratio of the frequency selective bandwidth f_0 to the chip rate R_c. The impulse response functions were generated using the frozen-in model ($C_{xt} = 1$ and $C_{yt} = 0$) and a small antenna (i.e., $D << \ell_0$). Each frame in the figure provides a three-dimensional picture of the matched-filter output amplitude for a single transmitted pulse as a function of time delay (abscissa) and time (scale directed into the figure). The total duration of each of the frames is 10 decorrelation times.

In the top frame the frequency selective bandwidth is equal to the chip rate and only a small amount of distortion is evident in the waveform (which is slightly rounded due to band limiting at the first nulls of the signal spectrum). The effect of fading can be seen in this frame as the peak amplitude rises and falls with time. Some minor distortion of the output amplitude is seen but for the most part the signal is contained within the period of one chip. This channel is nearly-flat fading which means that all frequency components within the signal bandwidth propagate essentially the same way through the disturbed ionosphere. There is very little time delay spread beyond one chip in the matched-filter output.

The middle frame in Figure 11 shows the matched-filter output amplitude for the case were f_0/R_c is equal to 0.2. For this smaller value of the frequency selective bandwidth, more of the signal energy is arriving with delays of more than a chip, and there are multiple distinct peaks in the matched-filter output amplitude. It is these structures that can cause delay tracking algorithms to lose lock and that cause intersymbol interference which can degrade demodulation performance.

The bottom frame shows a highly disturbed case where f_0 is one tenth of the chip rate. This causes signal energy to be spread over approximately eight chip periods. When a contiguous set of pulses is transmitted, the delay spread of the received signal results in the simultaneous reception of information from about eight previous chips that can produce severe intersymbol interference. An effect due to the frozen-in model that is evident in Figure 11 is that the signal arriving at long delays varies more rapidly in time than the signal arriving at shorter delays.

Examples of the channel transfer function at three different times (0, $\tau_0/2$, τ_0) are shown in Figure 12 for the case were f_0/R_c is equal to 0.1. The plots show the power of $H(\omega,t)$, in decibels, as a function of normalized frequency across the bandwidth. With $\Delta\tau$ equal to $T_c/2$, the unambiguous frequency range of the channel transfer function is from $-R_c$ to R_c so the abscissa in this plot is from −1 to 1.

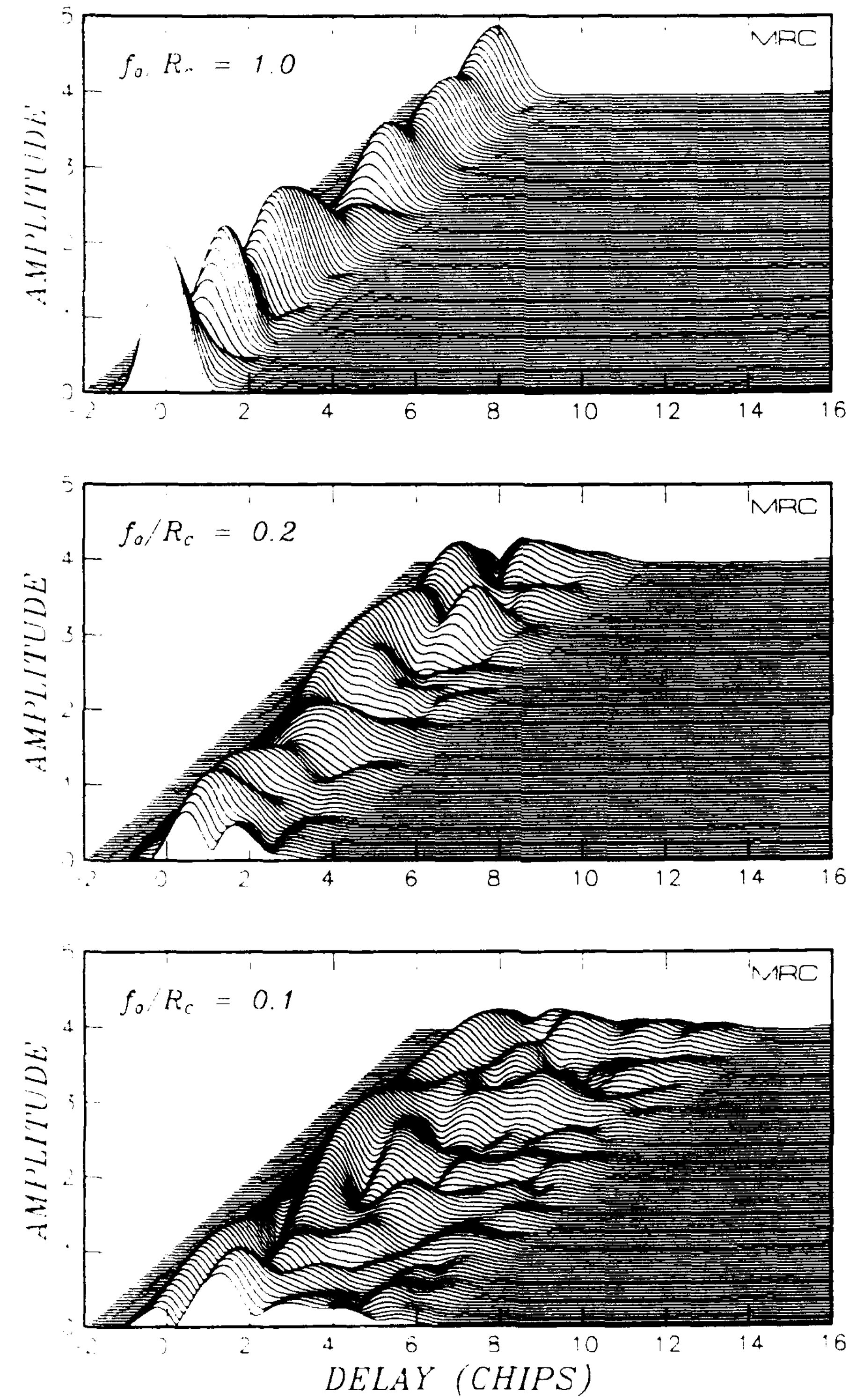

Figure 11. Effects of frequency selective fading on matched filter output amplitude.

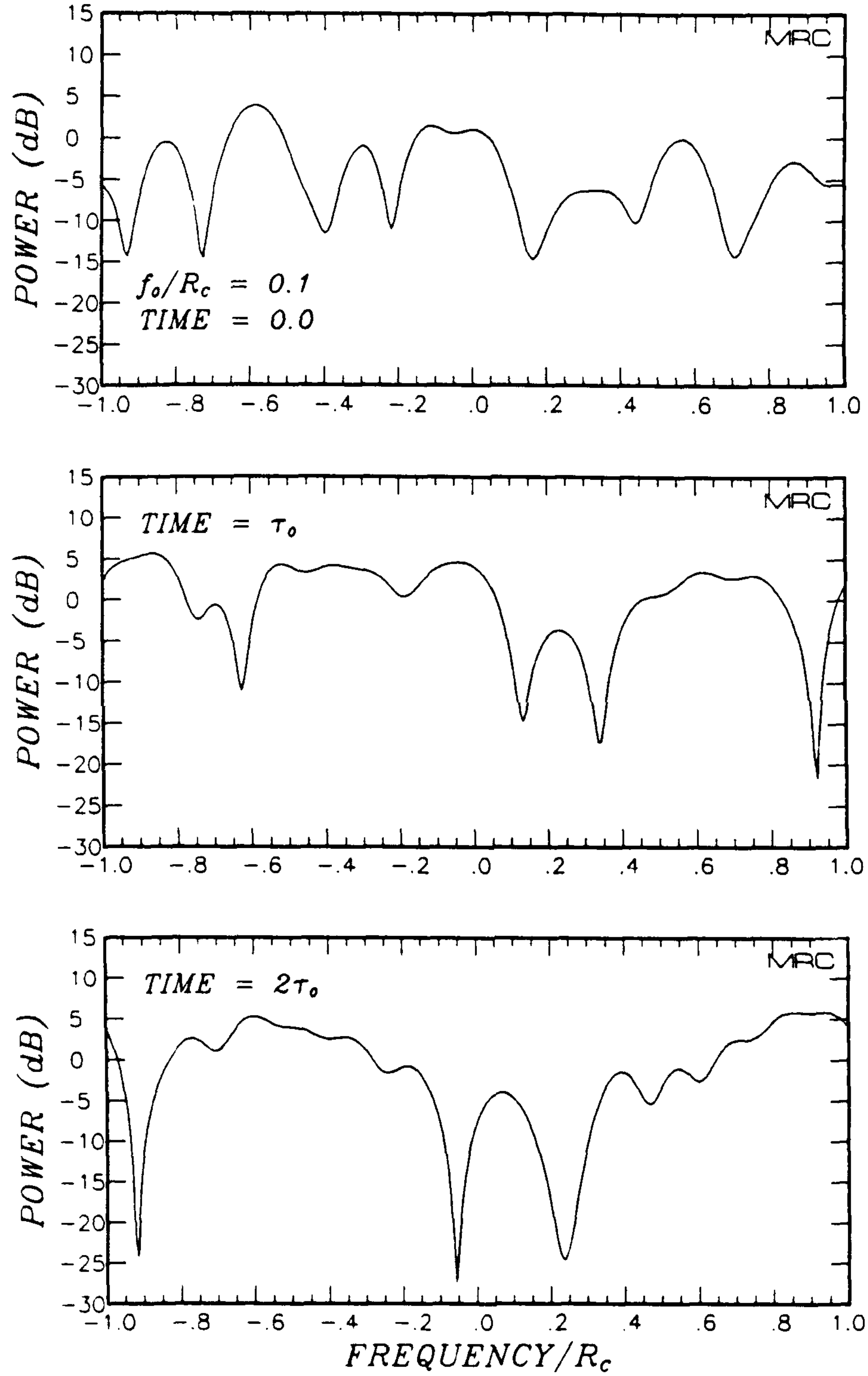

Figure 12. Amplitude of the channel transfer function at three times for the case f_0/R_c = 0.1.

A comparison of the matched-filter output amplitude generated with the frozen-in, general, and turbulent models is shown in Figure 13 for the case where f_0/R_c is equal to 0.1. The top frame in this figure for the frozen-in model is just a reproduction of the bottom frame in Figure 11. Again 10 decorrelation times of the signal are plotted. The middle frame is a general model realization ($C_{xt} = 0.9$ and $C_{yt} = 0$), and the bottom frame is for the turbulent model ($C_{xt} = C_{yt} = 0$). The difference between the top and bottom frames is that the turbulent model amplitude has the same fading rate at all delays. It can be seen that the general model realization falls somewhere between these two limiting cases.

4.3 SPATIALLY SELECTIVE EFFECTS.

Spatially selective effects are important for high data rate communications links that rely on large antennas to achieve sufficient signal-to-noise ratios for low error rate data demodulation. Scattering loss of an antenna is a function of the size of the antenna D relative to the decorrelation distance.

When ℓ_0 is greater than D the electric field is highly correlated across the face of the antenna and the full gain of the antenna is realized. However, the antenna may be located at a position where the incident power is in a deep fade. The solution to this problem is to have multiple antennas physically separated by a distance larger than the maximum decorrelation distance. The probability of having all antennas simultaneously experience deep fades in the received power is then substantially reduced.

The problem of spatial selectivity occurs when ℓ_0 is less than D and the electric field is decorrelated across the face of the antenna. In this case, the induced voltages in the antenna add noncoherently due to the random phase variations in the electric field, and a loss of signal power, or equivalently of antenna gain, is the result. From another perspective, this loss occurs when the angular scattering process responsible for amplitude and phase scintillation and frequency selective effects also causes some of the transmitted signal energy to be scattered out of the antenna beam.

Figure 14 shows examples of the matched-filter output amplitude for three levels of spatially selective propagation disturbance, characterized by the ratio of the antenna size to the decorrelation distance. The ratio of the frequency selective bandwidth to the chip rate is 0.1, and the antenna is pointing along the line-of-sight. The top frame is for the case where ℓ_0 is much greater than D, and is just a reproduction of the bottom frame of Figure 11. The middle frame is for a ℓ_0/D ratio of 0.5 where the scattering loss is 3.1 dB. The effect of the antenna is to attenuate preferentially the signal energy arriving at large angles and at large delays and thereby to reduce the delay spread of the output signal. In the bottom frame where ℓ_0/D is equal to 0.2, the output signal is almost flat with very little delay spread distortion of the matched-filter output. Although this substantially reduces the effects of frequency selective fading, the cost is an 8.8-dB reduction in the average signal power.

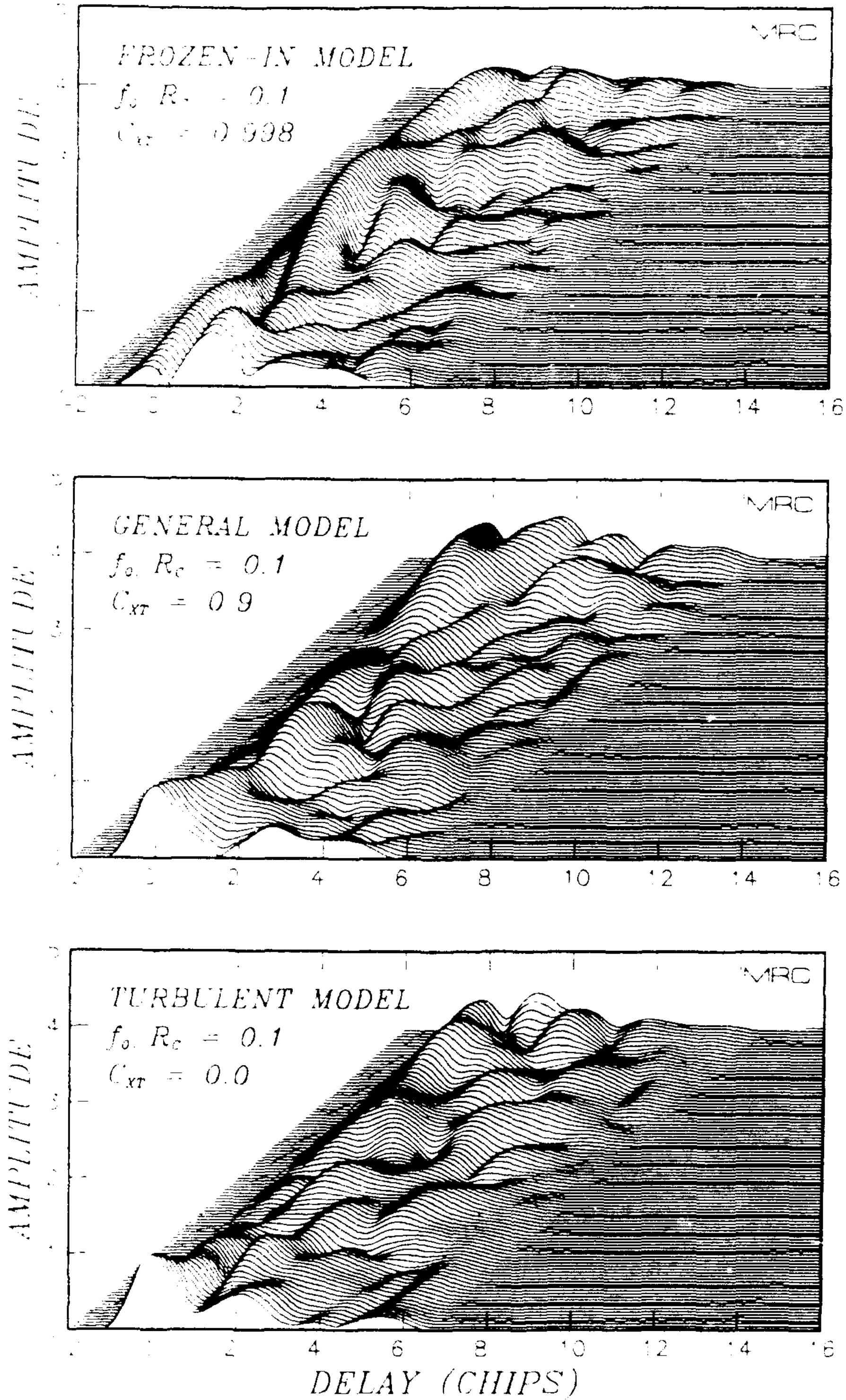

Figure 13. Comparison of matched filter output amplitude for the frozen-in, general, and turbulent models.

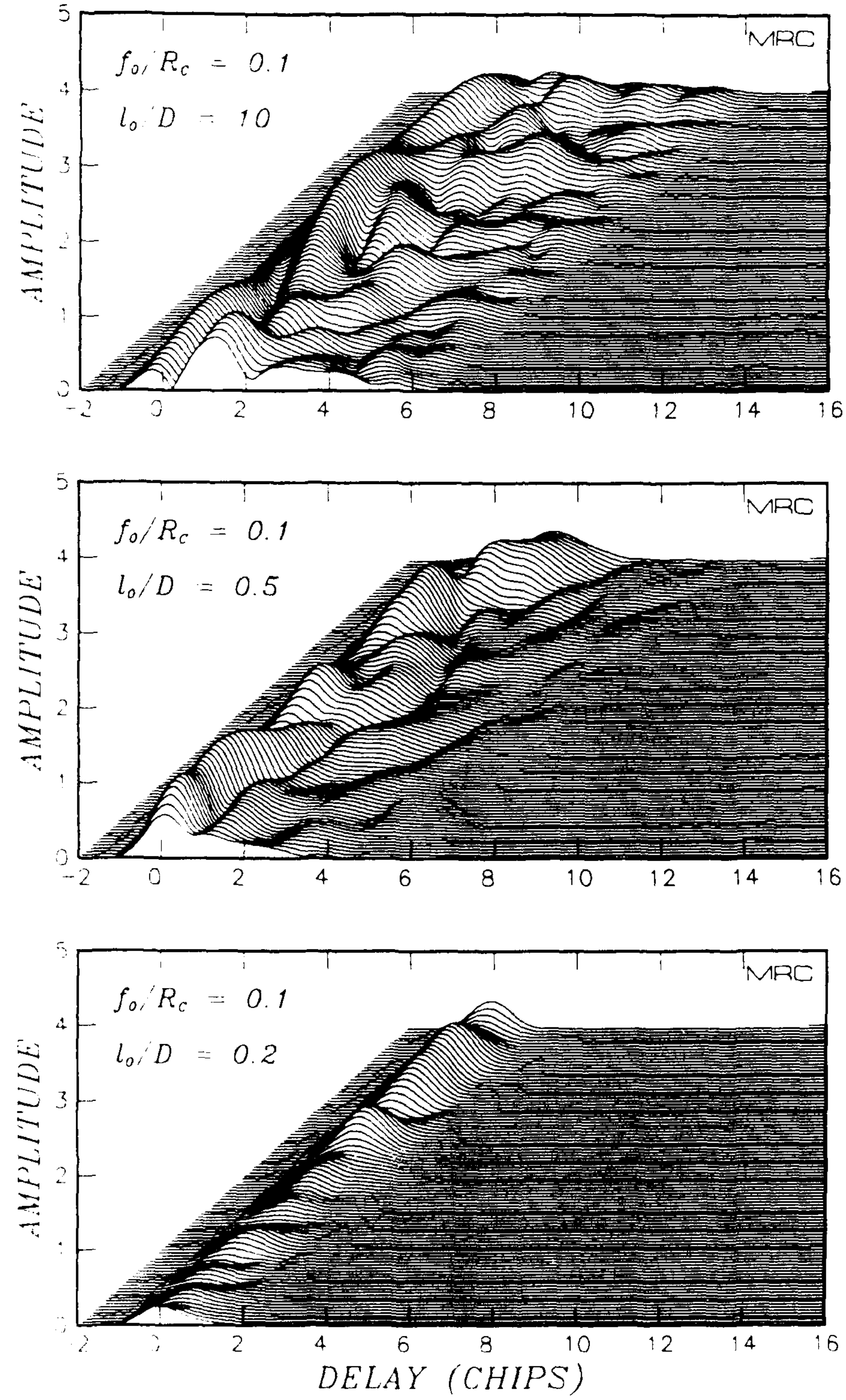

Figure 14. Effects of spatially selective fading on matched filter output amplitude.

Finally, Figure 15 shows examples of the matched-filter output amplitude for three values of the pointing angle Θ_0. The ratio of the frequency selective bandwidth to the chip rate is 0.1, and the ratio of decorrelation distance to antenna diameter is 0.5. The top frame for a pointing angle of zero is just a reproduction of the middle frame of Figure 14. The average scattering loss for this case is 3.1 dB. The bottom two frames show the matched-filter output amplitude for pointing angles of one-half beamwidth ($\Theta_0 = \theta_0/2$) with a scattering loss of 4.6 dB and one beamwidth ($\Theta_0 = \theta_0$) with a scattering loss of 9.2 dB.

Although the average scattering losses are about the same, the bottom frame of Figure 15 (ℓ_0/D = 0.5 and $\Theta_0 = \theta_0$) and the bottom frame of Figure 14 (ℓ_0/D = 0.2 and $\Theta_0 = 0$) are qualitatively quite different. For the case with the pointing angle equal to a beamwidth, the received power is much more spread out in delay compared to the case with zero pointing angle where the signal energy is concentrated near zero delay. This is because the antenna pointed away from the line-of-sight has relatively higher gain at large angles and long delays and relatively lower gain at small angles and short delays than does an antenna pointed along the line-of-sight. Thus for an antenna pointed away from the line-of-sight, increased scattering loss does not necessarily result in reduced frequency selective effects.

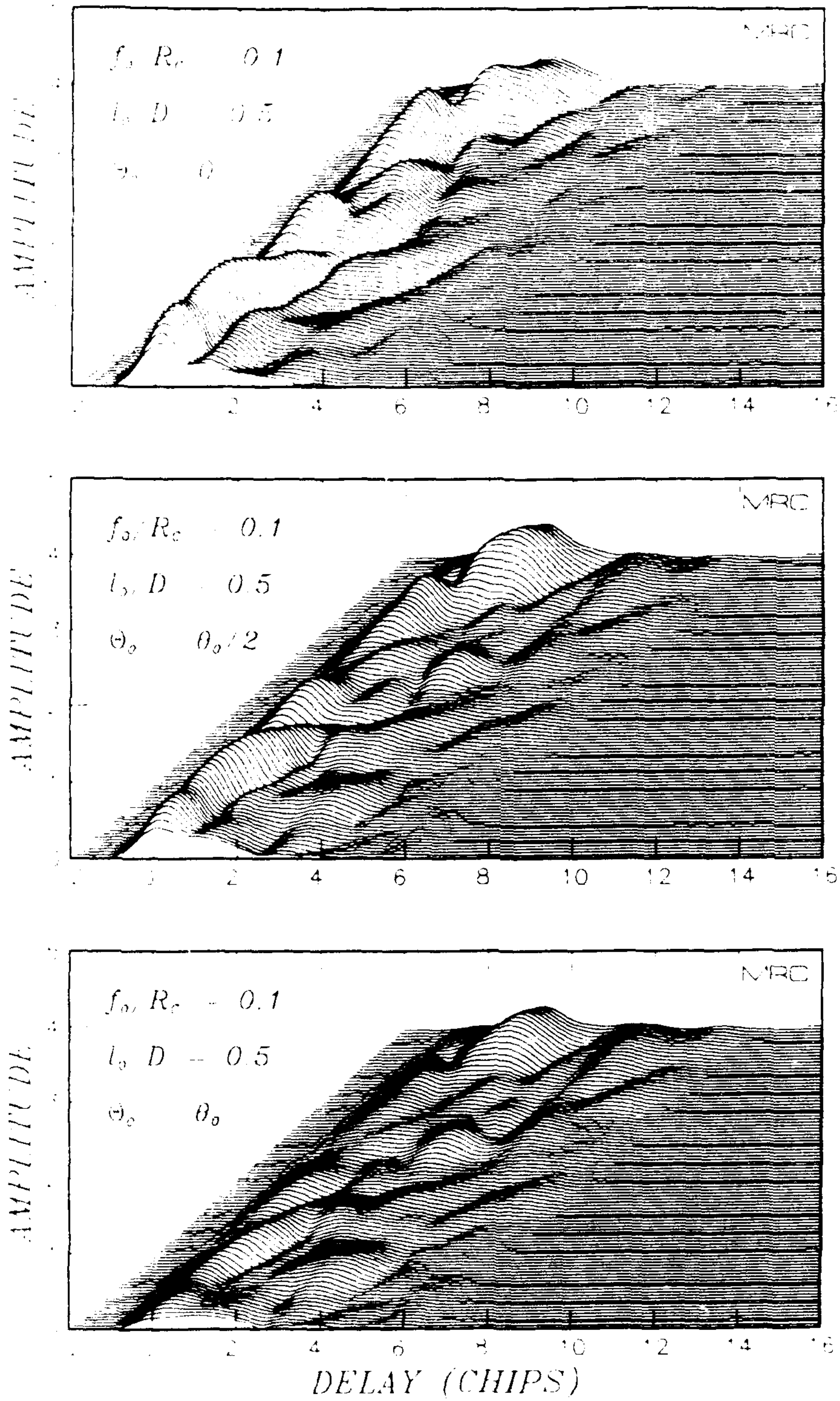

Figure 15. Effects of beam pointing on matched filter output amplitude.

SECTION 5
INPUTS AND OUTPUTS OF ACIRF AND CIRF

The latest version of ACIRF (version 3.5) is based on the general model with antenna aperture effects. A new version of CIRF (version 1.1) contains the older frozen-in and turbulent models without antenna effects. Both codes use a standard output file format. Complete listings of the two codes may be found in Appendix D.

Examples of ACIRF and CIRF input and output files are given in this section. For these examples, the codes were run on a Macintosh IIsi computer with a math coprocessor, the Motorola MC68882 floating-point unit. However the codes are written to produce machine independent output. Thus the user should be able to reproduce the output files on most 32 bit computers. When compared to the same case run on the Mission Research Corporation Elxsi 6400 computer, the Macintosh generated formatted output files from ACIRF or CIRF are identical, except for a few numbers that disagree in the 5th or 6th decimal place.

5.1 ACIRF INPUT AND FORMATTED OUTPUT FILES.

The key inputs to ACIRF are listed in Table 1. These include channel parameters that describe the second order statistics of the signal incident on the face of an antenna, antenna parameters, and realization parameters.

Relative antenna positions in the u–v coordinate system, u_0 and v_0, are antenna inputs along with beamwidths and pointing angles. The coordinates u_0 and v_0 are related to the antenna positions in the x–y coordinate system, x_0 and y_0, used in Equations 74 and 97 as follows:

$$x_0 = u_0 \cos\Psi - v_0 \sin\Psi \tag{132a}$$

$$y_0 = u_0 \sin\Psi + v_0 \cos\Psi \; . \tag{132b}$$

Key realization parameter inputs are the number of delay samples N_τ and the delay sample size $\Delta\tau$. The delay sample size should be no larger than one-half of the chip period. For most applications, setting $\Delta\tau$ equal to $T_c/2$ is sufficient. The number of delay samples required is determined by Equations 122 and 123. An estimate of the required value of the product $N_\tau\Delta\tau$ is given by Equation 126.

An example ACIRF input file is listed in Table 2. Note that comment lines in the input file start with a semicolon (;). This file is for a frozen-in model. Thus C_{xt} and C_{yt} are both slightly less than $\sqrt{2}$ so that $(C_{xt}^2 + C_{yt}^2)^{1/2}$ is slightly less than unity. Example ACIRF input and formatted output files for the turbulent model ($C_{xt} = C_{yt} = 0$) are listed in Appendix E.

Table 1. Key ACIRF inputs.

Channel Parameters	*Symbol*	*Units*
Frequency selective bandwidth	f_0	Hertz
Decorrelation distances	ℓ_x and ℓ_y	meters
Decorrelation time	τ_0	seconds
Space-time correlation coefficients	C_{xt} and C_{yt}	•
Antenna Parameters		
Beamwidths	θ_{0u} and θ_{0v}	degrees
Positions in u–v coordinate system	u_0 and v_0	meters
Rotation angle	Ψ	degrees
Pointing angle elevation	Θ_0	degrees
Pointing angle azimuth	Φ_0	degrees
Realization Parameters		
Number of delay samples	N_τ	•
Delay sample size	$\Delta\tau$	seconds
Number of time samples	N_T	•
Number of K_x samples	N_x	•
Number of K_y samples	N_y	•
Initial random number seed		•
Number of sample per decorrelation time	N_0	•

ACIRF version 3.5, dated 8 January 1990, contains some old notation from an earlier version of the code and *Dana* [1986]. In the input and formatted output files, CHI (Ψ in the notation used in this report and in *Dana* [1991]) is the rotation angle, and PSI (Θ_0 in this report) and EPS (Φ_0 in this report) are the antenna beam pointing elevation and azimuth angles, respectively. This inconsistency in notation has been corrected in ACIRF version 3.51, dated 25 April 1991, which was used to generated the example results in this report. In version 3.51, the rotation angle is denoted ROT, and ELV and AZM are the beam pointing elevation and azimuth angles, respectively.

The formatted output file generated using the example ACIRF data file is listed in Table 3. The first page of this file provides a summary of the inputs and lists, for each antenna, ensemble values of scattering loss and antenna filtered frequency selective bandwidth and decorrelation time. Subsequent pages of the output file list, for each antenna, measured values of these quantities. The last page lists ensemble and measured values for the cross correlation of the signal out of the antennas. The algorithms used to measure these quantities are given in *Dana* [1986].

Table 2. Example ACIRF input file for frozen-in model.

```
; GENERAL MODEL WITH ANTENNAS - FROZEN-IN MODEL LIMIT
;
;****** CASE NUMBER
;
;     KASE
      1001/
;
;****** ALPHANUMERIC IDENTIFICATION (80 CHARACTERS OR LESS ENCLOSED IN ' ')
;
'GENERAL MODEL (FROZEN-IN) WITH ANTENNAS - ACIRF 3.5 USERS GUIDE EXAMPLE'
;
;****** CHANNEL PARAMETERS
;
;   FO(HZ)   TAU0(S)    DLX(M)    DLY(M)     CXT      CYT
     1.0E5    3.0E-3       5.0       5.0    0.706   0.706/
;
;****** ANTENNA PARAMETERS
;
;   NUMANT FREQC(HZ)
         3 2.99793E9/
;
; BEAMWIDTHS, POSITIONS, ROTATION ANGLES, AND POINTING ANGLES
; (ONE LINE FOR EACH ANTENNA)
;   ROT = ROTATION ANGLE
;   ELV = ANTENNA BEAM POINTING ANGLE ELEVATION
;   AZM = ANTENNA BEAM POINTING ANGLE AZIMUTH
;
; BWU(DEG)  BWV(DEG)   UPOS(M)   VPOS(M)  ROT(DEG)  ELV(DEG)  AZM(DEG)
   0.5896    0.5896     -10.0       0.0      00.0    0.2948    180.0/
   0.5896    0.5896       0.0       0.0      00.0       0.0      0.0/
   0.5896    0.5896      10.0       0.0      00.0    0.2948      0.0/
;
;****** REALIZATION PARAMETERS
;
;   NDELAY DELTAU(S)    NTIMES       NKX       NKY     ISEED        NO
         8    5.0E-7      1024        32        32   9771975       10/
```

Table 3a. Example ACIRF formatted output file for the frozen-in model (summary of input and ensemble realization statistics).

```
ACIRF CHANNEL SIMULATION VERSION   3.51
  CASE NUMBER         1001
  TEMPORAL VARIATION FROM GENERAL MODEL
  REALIZATION IDENTIFICATION:
  GENERAL MODEL (FROZEN-IN) WITH ANTENNAS - ACIRF 3.5 USERS GUIDE EXAMPLE

CHANNEL PARAMETERS
  FREQUENCY SELECTIVE BANDWIDTH  (HZ) =  1.000E+05
  DECORRELATION TIME (SEC)            =  3.000E-03
  X DECORRELATION DISTANCE (M)        =  5.000E+00
  Y DECORRELATION DISTANCE (M)        =  5.000E+00
  TIME-X CORRELATION COEFFICIENT      =  7.060E-01
  TIME-Y CORRELATION COEFFICIENT      =  7.060E-01

REALIZATION PARAMETERS
  NUMBER OF DELAY SAMPLES             =          8
  DELAY SAMPLE SIZE (SEC)             =  5.000E-07
  NUMBER OF TEMPORAL SAMPLES          =       1024
  NUMBER OF DOPPLER FREQUENCY SAMPLES =        188
  NUMBER OF TEMPORAL SAMPLES PER TAU0 =         10
  NUMBER OF KX SAMPLES                =         32
  NUMBER OF KY SAMPLES                =         32
  INITIAL RANDOM NUMBER SEED        =      9771975

ANTENNA PARAMETERS
  NUMBER OF ANTENNAS       =          3
  CARRIER FREQUENCY (HZ)   =  2.998E+09

ANTENNA BEAMWIDTHS, ROTATION ANGLES AND POINTING ANGLES

  N   BWU(DEG)    BWV(DEG)    ROT(DEG)    ELV(DEG)    AZM(DEG)
  1      0.590       0.590       0.000       0.295     180.000
  2      0.590       0.590       0.000       0.000       0.000
  3      0.590       0.590       0.000       0.295       0.000

ANTENNA POSITIONS IN U-V AND X-Y COORDINATES

  N     UPOS(M)     VPOS(M)     XPOS(M)     YPOS(M)
  1 -1.000E+01  0.000E+00 -1.000E+01  0.000E+00
  2  0.000E+00  0.000E+00  0.000E+00  0.000E+00
  3  1.000E+01  0.000E+00  1.000E+01  0.000E+00

ENSEMBLE CHANNEL PARAMETERS AT ANTENNA OUTPUTS

  N   LOSS(DB)       POWER      FA(HZ)   TAUA(SEC)
  1      4.602    0.346611   1.574E+05   4.300E-03
  2      3.141    0.485167   2.061E+05   4.300E-03
  3      4.602    0.346611   1.574E+05   4.300E-03
```

Table 3b. Example ACIRF formatted output for the frozen-in model (summary of measured realization statistics for antenna 1).

MEASURED PARAMETERS FOR REALIZATION/ANTENNA 1

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
NORMALIZED TO ENSEMBLE VALUES
POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
J = T: STATISTICS OF COMPOSITE SIGNAL

J	POW(J)	<A>	<A^2>	<A^3>	<A^4>	S4	<CHI>	<CHI^2>
0	1.280E-01	0.9820	0.9624	0.9396	0.9042	0.9759	1.0275	0.9281
1	8.257E-02	0.8877	0.7869	0.6948	0.6111	0.9868	1.4130	1.1673
2	5.231E-02	1.0028	0.9724	0.9351	0.8976	0.9480	0.8641	0.7392
3	3.266E-02	1.0591	1.1461	1.2391	1.3290	1.0118	0.8932	1.1180
4	2.016E-02	1.0297	1.0477	1.0505	1.0362	0.9423	0.8680	0.8985
5	1.233E-02	0.9409	0.8885	0.8478	0.8185	1.0361	1.1933	1.0194
6	7.474E-03	1.0195	1.0252	1.0110	0.9734	0.9232	0.9111	0.9351
7	4.500E-03	1.0068	1.0458	1.1194	1.2389	1.1249	1.0501	1.1280
T	3.400E-01	0.9449	0.8889	0.8283	0.7635	0.9657	1.2082	1.1118

REALIZATION SIGNAL PARAMETERS:		ENSEMBLE	MEASURED
MEAN POWER OF REALIZATION	=	0.346611	0.320906
TOTAL SCATTERING LOSS (DB)	=	4.602	4.936
FREQUENCY SELECTIVE BANDWIDTH (HZ)	=	1.574E+05	1.847E+05
DECORRELATION TIME (SEC)	=	4.300E-03	3.746E-03
NUMBER OF SAMPLES PER DECORR. TIME	=	10	8.711

MEAN POWER IN GRID

POWER IN KX-KY GIRD	=	0.339619
POWER LOSS OF GRID (DB)	=	0.089
POWER IN DELAY GRID	=	0.339998

Table 3c. Example ACIRF formatted output for the frozen-in model (summary of measured realization statistics for antenna 2).

```
MEASURED PARAMETERS FOR REALIZATION/ANTENNA 2

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
  NORMALIZED TO ENSEMBLE VALUES
  POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
  J = T: STATISTICS OF COMPOSITE SIGNAL

  J     POW(J)      <A>    <A^2>    <A^3>    <A^4>       S4    <CHI> <CHI^2>
  0  2.313E-01   1.0121   1.0175   1.0295   1.0463   1.0105   0.9317   0.9732
  1  1.210E-01   0.9572   0.9471   0.9335   0.9052   1.0092   1.2895   1.3252
  2  6.334E-02   0.9658   0.9573   0.9535   0.9484   1.0344   1.2266   1.2665
  3  3.315E-02   1.0330   1.0579   1.0665   1.0562   0.9421   0.8832   0.9574
  4  1.735E-02   0.9468   0.8939   0.8396   0.7866   0.9842   1.1987   1.1069
  5  9.078E-03   0.9825   0.9636   0.9495   0.9358   1.0078   1.0315   0.9476
  6  4.751E-03   0.9286   0.8760   0.8306   0.7899   1.0290   1.3193   1.2396
  7  2.486E-03   1.0676   1.1543   1.2539   1.3632   1.0229   0.8109   0.9955
  T  4.824E-01   0.9790   0.9627   0.9540   0.9483   1.0229   1.0790   1.0466

REALIZATION SIGNAL PARAMETERS:                 ENSEMBLE        MEASURED

  MEAN POWER OF REALIZATION             =    0.485167        0.476912
  TOTAL SCATTERING LOSS (DB)            =       3.141           3.216
  FREQUENCY SELECTIVE BANDWIDTH (HZ)    =   2.061E+05       2.289E+05
  DECORRELATION TIME (SEC)              =   4.300E-03       4.260E-03
  NUMBER OF SAMPLES PER DECORR. TIME    =          10           9.906

MEAN POWER IN GRID

  POWER IN KX-KY GIRD      =   0.482493
  POWER LOSS OF GRID (DB) =       0.024
  POWER IN DELAY GRID      =   0.482437
```

Table 3d. Example ACIRF formatted output for the frozen-in model (summary of measured realization statistics for antenna 3).

```
MEASURED PARAMETERS FOR REALIZATION/ANTENNA 3

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
  NORMALIZED TO ENSEMBLE VALUES
  POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
  J = T: STATISTICS OF COMPOSITE SIGNAL

  J     POW(J)      <A>     <A^2>    <A^3>    <A^4>        S4    <CHI> <CHI^2>
  0  1.280E-01  1.0975  1.1489  1.1529  1.1154  0.8307  0.5969  0.8006
  1  8.257E-02  1.0582  1.1536  1.2672  1.3684  1.0279  0.8453  0.9614
  2  5.231E-02  0.9523  0.9132  0.8904  0.8766  1.0498  1.1477  0.9950
  3  3.266E-02  1.0784  1.1398  1.1978  1.2556  0.9659  0.6671  0.7841
  4  2.016E-02  0.9340  0.8511  0.7763  0.7216  0.9962  1.1575  0.9373
  5  1.233E-02  0.9827  0.9417  0.8788  0.8048  0.9028  1.0295  0.9829
  6  7.474E-03  1.0027  1.0233  1.0547  1.0836  1.0343  1.0234  1.0476
  7  4.500E-03  0.9420  0.8578  0.7616  0.6598  0.8907  1.1216  0.9100
  T  3.400E-01  1.0769  1.1606  1.2236  1.2517  0.9266  0.7919  1.0408

REALIZATION SIGNAL PARAMETERS:                 ENSEMBLE       MEASURED

  MEAN POWER OF REALIZATION            =   0.346611       0.367584
  TOTAL SCATTERING LOSS (DB)           =      4.602          4.346
  FREQUENCY SELECTIVE BANDWIDTH (HZ)   =  1.574E+05      1.965E+05
  DECORRELATION TIME (SEC)             =  4.300E-03      4.500E-03
  NUMBER OF SAMPLES PER DECORR. TIME   =         10         10.466

MEAN POWER IN GRID

  POWER IN KY-KY GIRD      =   0.339502
  POWER LOSS OF GRID (DB) =      0.090
  POWER IN DELAY GRID      =   0.339998
```

Table 3e. Example ACIRF formatted output for the frozen-in model (ensemble and measured antenna output cross correlation coefficients).

```
ENSEMBLE ANTENNA OUTPUT CROSS CORRELATION

  AMPLITUDE OF CROSS CORRELATION

   N     AMP(N-1)   AMP(N-2)   AMP(N-3)
   1     1.000000   0.131351   0.000298
   2     0.131351   1.000000   0.131351
   3     0.000298   0.131351   1.000000

  PHASE (RADIANS) OF CROSS CORRELATION

   N     PHS(N-1)   PHS(N-2)   PHS(N-3)
   1     0.000000  -0.832184   0.000000
   2    -0.832184   0.000000   0.832184
   3     0.000000   0.832184   0.000000


MEASURED ANTENNA OUTPUT CROSS CORRELATION

  AMPLITUDE OF CROSS CORRELATION

   N     AMP(N-1)   AMP(N-2)   AMP(N-3)
   1     1.000000   0.175555   0.065979
   2     0.175555   1.000000   0.236165
   3     0.065979   0.236165   1.000000

  PHASE (RADIANS) OF CROSS CORRELATION

   N     PHS(N-1)   PHS(N-2)   PHS(N-3)
   1     0.000000  -2.194468   0.274303
   2    -2.194468   0.000000   2.185150
   3     0.274303   2.185150   0.000000

FINAL RANDOM NUMBER SEED =    -101148759
```

For each delay bin, moments of the amplitude, the scintillation index S_4, and moments of the log amplitude of the impulse response function are measured and output in the formatted file. The ensemble values for the amplitude moments are given below. The expected variation of the measured values of these parameters about their ensemble values is given in Appendix C.

A realization of the impulse response function at a given delay is a complex, zero-mean, normally distributed random variable. Thus the amplitude of the realization is Rayleigh distributed. Close agreement of the measured amplitude moments and S_4 with their ensemble values indicates that the flares (i.e., times when the instantaneous power is about equal to or exceeds the mean power) in the realization closely follow a Rayleigh distribution. Close agreement of the of the log amplitude moments indicates that the fades closely follow a Rayleigh distribution.

Because different realizations will have significant differences in the measured values of the moments of the amplitude, log amplitude, and scintillation index, a comparison of these moments can be used to quickly determine whether two realizations are of the impulse response function are identical. Even minor variations in a realization may result in large variations in the moments. This is particularly true for short (100 decorrelation times) realizations.

Ensemble values of the amplitude moments depend on the ensemble power in the j^{th} delay bin for the m^{th} antenna:

$$P_{j,m} = \int_{j\Delta\tau}^{(j+1)\Delta\tau} S_{A,m}(\tau)\, d\tau \quad . \qquad (j = 0, 1, \cdots, N_\tau - 1) \tag{133}$$

The delay power spectral density $S_{A,m}(\tau)$ at the output of the m^{th} antenna pointed in the direction $\boldsymbol{K}_{0,m}$ with beam profile $G_m(\boldsymbol{K}_\perp)$ is:

$$S_{A,m}(\tau) = \int_{-\infty}^{\infty} G_m(\boldsymbol{K}_\perp - \boldsymbol{K}_{0,m})\, S_{K\tau}(\boldsymbol{K}_\perp, \tau)\, \frac{d\boldsymbol{K}_\perp}{(2\pi)^2} \quad . \tag{134}$$

A closed-form expression for $S_{A,m}(\tau)$ is given by *Frasier* [1990].

Ensemble values of the moments of the amplitude $\langle a_j^n \rangle$, log amplitude $\langle \chi^n \rangle$, and the scintillation index S_4 are:

$$\langle a_j \rangle = \frac{1}{2}\sqrt{\pi P_{j,m}} \tag{135a}$$

$$\langle a_j^2 \rangle = P_{m,j} \tag{135b}$$

$$\langle a_j^3 \rangle = \frac{3}{4}\sqrt{\pi P_{j,m}^3} \tag{135c}$$

$$\langle a_j^4 \rangle = 2P_{j,m}^2 \tag{135d}$$

$$S_4 = \left[\frac{\langle a_j^4 \rangle - \langle a_j^2 \rangle}{\langle a_j^2 \rangle}\right]^{\frac{1}{2}} = 1 \tag{135e}$$

$$\langle \chi \rangle = \langle \ln(a_j) \rangle = \frac{1}{2}\ln(P_{j,m}) - \frac{\gamma}{2} \tag{135f}$$

$$\langle \chi^2 \rangle = \langle \ln^2(a_j) \rangle = \frac{1}{4}\ln^2(P_{j,m}) - \frac{\gamma}{2}\ln(P_{j,m}) + \frac{1}{4}\left(\frac{\pi^2}{6} + \gamma^2\right) \tag{135g}$$

where γ is Euler's constant ($\gamma = 0.5772157\cdots$).

To assure the user that the angle-delay grids contain most the signal energy, the mean power in these grids is an output of ACIRF. The mean power in the $K_x - K_y$ grid, for the m^{th} antenna, is

$$P_{K,m} = \sum_{k_x=-N_x/2}^{N_x/2-1} \sum_{k_y=-N_y/2}^{N_y/2-1} E_m(k_x,k_y) \ , \tag{136}$$

where $E_m(k_x,k_y)$ is the mean power in a $K_x - K_y$ grid cell. This quantity is discussed further in Appendix B. The mean power in the delay gird, for the m^{th} antenna, is

$$P_{\tau,m} = \sum_{j=0}^{N_\tau - 1} P_{j,m} \ . \tag{137}$$

These quantities may be found in the ACIRF formatted output under the title "MEAN POWER IN GRID". The loss associated with $P_{K,m}$ is the ratio of $P_{K,m}$ to the scattering loss of the m^{th} antenna. This loss, in decibels, is also in the formatted ACIRF output file.

5.2 CIRF INPUT AND FORMATTED OUTPUT FILES.

The key inputs to CIRF are listed in Table 4. These include channel parameters that describe the second order statistics of the signal and realization parameters. The models in CIRF do not include the effects of antennas. Thus the GPSD used in these

models is integrated over K_x and K_y, so decorrelation distances are not required. The frequency selective model in CIRF is generated assuming isotropic scattering.

There are four different channel models included in CIRF, all without antenna effects: an additive white Gaussian noise channel (IFADE = 0); a frequency selective frozen-in model (IFADE = 2); a frequency selective turbulent model (IFADE = 3); and a flat fading model (IFADE = 4). Models 2 and 4 were also implemented in Wittwer's original CIRF program [*Wittwer*, 1980]. The general model with antenna effects contained in ACIRF corresponds to the IFADE = 1 case and is not allowed in CIRF.

Key realization parameter inputs are the number of delay samples N_τ and the delay sample size $\Delta\tau$. The delay sample size should be no larger than one-half of the chip period. For most applications, setting $\Delta\tau$ equal to $T_c/2$ is sufficient. Equation 126, with f_A replaced by f_0, also gives the required extent, $N_\tau\Delta\tau$, of the delay grid for the frequency selective models in CIRF [*Dana*, 1986; *Wittwer*, 1980]:

$$N_\tau \Delta\tau = \frac{3.7}{2\pi f_0} \ . \tag{138}$$

If $\Delta\tau$ is fixed, this equation gives the required number of delay samples.

An example CIRF input file for the frozen-in model is listed in Table 5. Comment lines in the input file start with a semicolon (;). Example input and formatted files for the other three channel models in CIRF are listed in Appendix E.

The formatted output file generated using the example CIRF data file is listed in Table 6. The first page of this file provides a summary of the inputs. The second page of the output file lists moments of the amplitude, the scintillation index S_4, and moments of the log amplitude for each delay bin. The mean power in the Doppler-delay grids is also printed.

Table 4. Key CIRF inputs.

Channel Parameters	*Symbol*	*Units*
Channel model	IFADE	
Frequency selective bandwidth	f_0	Hertz
Decorrelation time	τ_0	seconds
Realization Parameters		
Number of delay samples	N_τ	•
Delay sample size	$\Delta\tau$	seconds
Number of time samples	N_T	•
Initial random number seed		•
Number of sample per decorrelation time	N_0	•

Table 5. Example CIRF input file for frozen-in model.

```
; FREQUENCY SELECTIVE FADING - FROZEN-IN MODEL FOR TEMPORAL FLUCTUATIONS
;
;****** CASE NUMBER
;
;     KASE
      2002/
;
;****** ALPHANUMERIC IDENTIFICATION (80 CHARACTERS OR LESS ENCLOSED IN ' ')
;
'FROZEN-IN MODEL - ACIRF 3.5 USERS GUIDE EXAMPLE'
;
;****** CHANNEL PARAMETERS
;
;    IFADE     FO(HZ)    TAUO(S)
         2      1.0E5     3.0E-3/
;
;****** REALIZATION PARAMETERS
;
;   NDELAY DELTAU(S)     NTIMES     ISEED        NO
        11     5.0E-7      1024   9771975        10/
```

Table 6a. Example CIRF output file for frozen-in model (page 1, summary of input and ensemble values).

```
 CIRF CHANNEL SIMULATION VERSION   1.11
   CASE NUMBER          2002
   TEMPORAL VARIATION FROM FROZEN-IN MODEL
   REALIZATION IDENTIFICATION:
   FROZEN-IN MODEL - ACIRF 3.5 USERS GUIDE EXAMPLE

 CHANNEL PARAMETERS
   FREQUENCY SELECTIVE BANDWIDTH  (HZ) =  1.000E+05
   DECORRELATION TIME (SEC)            =  3.000E-03

 REALIZATION PARAMETERS
   NUMBER OF DELAY SAMPLES             =         11
   DELAY SAMPLE SIZE (SEC)             =  5.000E-07
   NUMBER OF TEMPORAL SAMPLES          =       1024
   NUMBER OF TEMPORAL SAMPLES PER TAUO =         10
   INITIAL RANDOM NUMBER SEED        =      9771975
```

Table 6b. Example CIRF output file for frozen-in model (page 2, summary of measured realization statistics).

MEASURED PARAMETERS OF REALIZATION

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
NORMALIZED TO ENSEMBLE VALUES
POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
J = T: STATISTICS OF COMPOSITE SIGNAL

J	POW(J)	<A>	<A^2>	<A^3>	<A^4>	S4	<CHI>	<CHI^2>
0	4.741E-02	0.8220	0.6857	0.5702	0.4673	0.9937	1.7158	1.3564
1	1.546E-01	0.9814	0.9265	0.8549	0.7728	0.8946	0.9582	0.8257
2	2.008E-01	1.0682	1.1600	1.2627	1.3608	1.0113	0.8206	1.0239
3	1.631E-01	1.0600	1.0881	1.0804	1.0394	0.8694	0.7607	0.9237
4	1.189E-01	0.9882	0.9792	0.9619	0.9288	0.9682	1.0704	1.0935
5	8.600E-02	1.0018	0.9933	0.9716	0.9323	0.9432	0.9474	0.8705
6	6.221E-02	1.0038	1.0011	0.9953	0.9883	0.9861	0.9735	0.9756
7	4.500E-02	1.0302	1.0218	0.9907	0.9456	0.9008	0.8186	0.8659
8	3.255E-02	1.0216	1.0410	1.0480	1.0442	0.9628	0.9720	1.1175
9	2.355E-02	1.0649	1.1107	1.1533	1.1945	0.9678	0.7005	0.7707
10	1.703E-02	1.0255	1.0565	1.0874	1.1165	1.0002	0.9644	1.1319
T	9.510E-01	0.9408	0.9127	0.8906	0.8688	1.0420	1.3307	1.3519

REALIZATION SIGNAL PARAMETERS:		ENSEMBLE	MEASURED
MEAN POWER OF REALIZATION	=	1.000000	0.974180
LOSS (DB) DUE TO MEAN POWER	=	0.000	0.114
FREQUENCY SELECTIVE BANDWIDTH (HZ)	=	1.000E+05	1.368E+05
DECORRELATION TIME (SEC)	=	3.000E-03	2.771E-03
NUMBER OF SAMPLES PER DECORR. TIME	=	10	9.237

MEAN POWER IN GRID

POWER IN DOPPLER GIRD = 0.949248
POWER LOSS OF GRID (DB) = 0.226
POWER IN DELAY GRID = 0.951048

FINAL RANDOM NUMBER SEED = -1505951035

5.3 UNFORMATTED ACIRF AND CIRF OUTPUT FILES.

The structure of the unformatted ACIRF and CIRF output files that contain the impulse response function realizations is described in Table 7. The first record of an unformatted file is the 80 character identification that is an ACIRF input. The Fortran code that writes this record is shown in Table 8.

The problem definition data record A contains a summary of the input data and record B contains a detailed definition of the realizations. The generating Fortran code is listed in Table 9. A description of the important words in record A is given in Table 10. Because a common output file format is used by several channel simulation programs developed at Mission Research Corporation, ACIRF and CIRF do not use all the words in the record. Only the words that contain information relevant to the use of ACIRF or CIRF realizations are listed in Table 10. The second column of Table 10 indicates which variables are used by CIRF. All other words in record A are zeros.

One of the key variables listed in Table 10 has not yet been defined. This is TAUMIN that is the minimum starting delay. The GPSD used in CIRF for the frozen-in model is given by Equations 11 and 12a with a non-infinite value of the parameter α (α = 4 is used) and isotropic scattering ($\ell_x = \ell_y = \ell_0$). Once the decorrelation distances are equated, the GPSD is integrated over K_x and K_y. The resulting delay-Doppler GPSD is then independent of ℓ_0. For a finite value of α, the starting delay is negative and is given by the formula

$$\tau_{min} = -\frac{0.25}{2\pi f_0} \tag{139}$$

as described in *Wittwer* [1980] or *Dana* [1986]. For the general model in ACIRF and for the turbulent model in CIRF, the value of α is infinite, and the minimum delay is zero.

Table 7. Structure of unformatted ACIRF and CIRF output files.

Record	*Description*
1	Character identification record
2	Floating point record A (problem definition data)
3	Floating point record B (detailed problem definition data)
4	Record A
5	Floating point realization data record 1
6	Record A (if necessary)
7	Floating point realization data record 2 (if necessary)
•	•
•	•
•	•
$2n+2$	Record A (if necessary)
$2n+3$	Floating Point realization data record n (if necessary)

Table 8. Fortran code that generates character identification record.

```
PARAMETER (NDENT=80)
CHARACTER IDENT*80
WRITE(IUNIT) NDENT,IDENT(1:NDENT)
```

Table 9. Fortran code that generates floating point header records A and B.

```
PARAMETER (NDATA1=30,NDATA2=32)
COMMON/HEADER/RDATA1(NDATA1),RDATA2(NDATA2)
WRITE(IUNIT) NDATA1,(RDATA1(II),II=1,NDATA1)
WRITE(IUNIT) NDATA2,(RDATA2(II),II=1,NDATA2)
```

Table 10. Description of header record A.

Word No.	*Fortran Variable*	*CIRF Variable*	*Parameter*
1	2.0	Yes	ACIRF or CIRF realization
2	KASE	Yes	Case number
3	FREQC	No	Carrier frequency (Hz)
4	TAU0	Yes	Decorrelation time τ_0 (sec)
5	F0	Yes	Frequency selective bandwidth f_0 (Hz)
6	DLX	No	x–decorrelation distance ℓ_x (m)
7	DLY	No	y–decorrelation distance ℓ_y (m)
9	1.0	Yes	Scintillation index S_4
13	TDUR	Yes	Time duration of realization ($N_T \Delta t$)
14	NTIMES	Yes	Number of time samples N_T
15	DELTAT	Yes	Time sample size Δt
16	N0	Yes	Samples per decorrelation time N_0
20	NDELAY	Yes	Number of delay samples N_τ
21	TAUMIN	Yes	Minimum delay
22	DELTAU	Yes	Delay sample size $\Delta\tau$
23	ISEED	Yes	Random number seed
24	JSEED	Yes	Random number seed
25	MAXBUF	Yes	Maximum buffer size
28	VERSON	Yes	Version No. of ACIRF/CIRF code

The ACIRF and CIRF versions of the subroutines WRITER and READER that write and read the unformatted files, respectively, are listed in Appendix D. The subroutine READER may be adapted to read the impulse response function into a link simulation. As currently written, READER outputs, through the call statement, an array containing the impulse response function $h(j\Delta\tau, k_T\Delta t)$ ($j = 1, \cdots, N_\tau$) at a fixed time sample $k_T\Delta t$ ($k_T = 1, \cdots, N_T$). Consecutive calls to READER produce consecutive time samples of the impulse response function.

The subroutine WRITER is called once each time step $k_T\Delta t$ ($k_T = 1, \cdots, N_T$) and passed an array of the impulse response function $h(j\Delta\tau, k_T\Delta t)$ ($j = 1, \cdots, N_\tau$). The subroutine WRITER buffers these arrays until the end of the realization is reached or until the number of impulse response function arrays equals the capacity of the buffer.

The maximum number of arrays in buffer is equal to the largest integer that is less than or equal to (MAXBUF/2)/NDELAY. The parameter MAXBUF is the maximum number of *real* words in an unformatted file record. The current value of MAXBUF is 4096. Once the buffer is filled to capacity or the end of the realization is reached, WRITER writes the buffer into the file as a single record.

The subroutine READER reverses this process. It first reads a record into a buffer, and then outputs a single impulse response function array from the buffer each time it is called. After (MAXBUF/2)/NDELAY calls to READER, another record is read from the file into the buffer, and so on.

SECTION 6
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APPENDIX A
ACCURACY OF ANGULAR INTEGRATION TECHNIQUES

The purpose of this appendix is to compute the accuracy of an approximation used in ACIRF to the angular K_x and K_y integrals in Equation 81. There are several ways that the angular integrals can be simplified, as reported in *Dana* [1989, 1991]. (Note that the error curves for the algorithm denoted $G(K)S(K)\Delta K$ in these two references are in error.) Only the accuracy of the approximation to Equation 81 actually used in ACIRF will be considered in this appendix.

A.1 INTRODUCTION.

The mean signal power in a $K_x - K_y$ grid cell at the output of an antenna is

$$E_A(k_x,k_y) = \int_{(k_x - 1/2)\Delta K_x}^{(k_x + 1/2)\Delta K_x} \frac{dK_x}{2\pi} \int_{(k_y - 1/2)\Delta K_y}^{(k_y + 1/2)\Delta K_y} \frac{dK_y}{2\pi} G(\boldsymbol{K}_\perp - \boldsymbol{K}_0)\, S_K(\boldsymbol{K}_\perp) \tag{140}$$

where $S_K(\boldsymbol{K}_\perp)$ is the angular part of the GPSD and $G(\boldsymbol{K}_\perp - \boldsymbol{K}_0)$ is the antenna beam profile for a beam pointing in the direction $\boldsymbol{K}_0$. Equation 140 is quite general, but it requires that the power in each angular bin be calculated and stored for each antenna or beam pointing direction. This latter requirement results in unacceptably large arrays. However, Equation 140 can be approximated by assuming the antenna beam pattern varies slowly over the $K_x - K_y$ grid cells so the antenna beam profile $G(\boldsymbol{K}_\perp - \boldsymbol{K}_0)$ may be pulled out of the integral.

To limit the scope of this calculation, isotropic scattering and a uniformly-weighted circular antenna will be assumed. Without further loss of generality, it can then be assumed that the antenna is pointed away from the line-of-sight in the x-direction, or equivalently, that both the pointing azimuth and rotation angles are zero. For this case, the angular part of the GPSD is

$$S_K(\boldsymbol{K}_\perp) = \pi \ell_0^2 \exp\left[- \frac{(K_x^2 + K_y^2)\ell_0^2}{4} \right] \tag{141}$$

where ℓ_0 is the isotropic decorrelation distance ($\ell_x = \ell_y = \ell_0$). The isotropic antenna beam pattern is

$$G(\boldsymbol{K}_\perp - \boldsymbol{K}_0) = \exp\left[- \frac{(Q-1)(K_x - K_0)^2 \ell_0^2}{4} - \frac{(Q-1)K_y^2 \ell_0^2}{4} \right] \tag{142}$$

where K_0 is the magnitude of the vector $\boldsymbol{K}_0$. The quantity $Q - 1$ is proportional to the square of the ratio of the antenna diameter D to the decorrelation distance:

$$Q = 1 + \frac{4\ln 2}{(a_0\pi)^2}\frac{D^2}{\ell_0^2} \tag{143}$$

where $a_0 = 1.02899$ for uniformly weighted circular antennas.

The antenna-filtered decorrelation distance, which is the same in both the x- and y-directions, is

$$\ell_A = \ell_0\sqrt{Q} \ . \tag{144}$$

This expression is then used in Equations 108 to compute the maximum required extent of the angular grids (Eqn. 109). When the antenna is pointed in the K_x direction, the K_x angular grid size is given by the expression

$$\Delta K_x = \frac{2}{N_x}\left[\frac{(Q-1)K_0}{Q} + \frac{\kappa_{K,max}}{\ell_0\sqrt{Q}}\right] \ , \tag{145a}$$

and the K_y angular grid size is

$$\Delta K_y = \frac{2}{N_y}\left[\frac{\kappa_{K,max}}{\ell_0\sqrt{Q}}\right] \ . \tag{145b}$$

The parameter $\kappa_{K,max}$ (Eqn. 103b) is determined by the condition that 99.9 percent of the signal energy be in the K_x–K_y grid, and N_x and N_y are the number of angular grid cells in the K_x and K_y directions, respectively.

The exact expression for the received power will be compared to the total power in the grid,

$$P_K = \sum_{k_x=-N_x/2}^{N_x/2-1}\ \sum_{k_y=-N_y/2}^{N_y/2-1} E_A(k_x,k_y) \ , \tag{146}$$

to compute an error in the total power

$$Error = \left|\frac{P_K - 0.999P_A}{0.999P_A}\right| \tag{147}$$

for each of the algorithms used to evaluate Equation 140. For this isotropic scattering and antenna case, the power at the output of the antenna P_A is given by Equation 52. The factor 0.999 occurs in this equation because the grid is sized to encompass this fraction of the total power.

A.2 ALGORITHMS.

The first approximation to the exact result is just Equation 140. This "exact" expression will result in some error in the total power because of inaccuracy in computing the error functions in the expression below and because of round-off errors in the summation of the contributions from each grid cell.

With the assumptions of isotropic scattering and an isotropic Gaussian beam pattern, the integrals indicated in Equation 140 can be obtained in closed form with the result

$$E_1(k_x,k_y) = \frac{P_A}{4}\left\{\mathrm{erfc}\left[\frac{(k_x-1/2)\Delta K_x \ell_0\sqrt{Q}}{2} - \frac{(Q-1)K_0\ell_0}{2\sqrt{Q}}\right]\right. \tag{148}$$

$$\left. - \mathrm{erfc}\left[\frac{(k_x+1/2)\Delta K_x \ell_0\sqrt{Q}}{2} - \frac{(Q-1)K_0\ell_0}{2\sqrt{Q}}\right]\right\}$$

$$\times \left\{\mathrm{erfc}\left[\frac{(k_y-1/2)\Delta K_y \ell_0\sqrt{Q}}{2}\right] - \mathrm{erfc}\left[\frac{(k_y+1/2)\Delta K_y \ell_0\sqrt{Q}}{2}\right]\right\}$$

where

$$K_0\ell_0 = 2\pi a_0 \frac{\Theta_0}{\theta_0}\frac{\ell_0}{D} , \tag{149}$$

Θ_0 is the pointing direction elevation angle, and θ_0 is the beamwidth of the antenna. The complementary error functions, erfc $(\cdot)$, in Equation 149 are computed in this analysis and in ACIRF using the most accurate rational approximation for the error function in *Abramowitz and Stegun* [1972].

The approximation used to the simplify the angular integrals in Equation 81 in ACIRF is obtained by assuming the antenna beam pattern is constant over an angular grid cell and can therefore be pulled out of the integral. The result is the product of the antenna beam pattern times a term that is equal to the incident power in a grid cell:

$$E_2(k_x,k_y) = G(k_x\Delta K_x - K_0, k_y\Delta K_y)\, E_I(k_x,k_y) \tag{150}$$

where the incident power in a grid cell is

$$E_I(k_x,k_y) = \int_{(k_x-1/2)\Delta K_x}^{(k_x+1/2)\Delta K_x} \frac{dK_x}{2\pi} \int_{(k_y-1/2)\Delta K_y}^{(k_y+1/2)\Delta K_y} \frac{dK_y}{2\pi} S_K(K_x,K_y) . \tag{151}$$

The indicated integrals can again be expressed in terms of error functions:

$$E_l(k_x,k_y) = \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{(k_x-1/2)\Delta K_x \ell_0}{2}\right] - \mathrm{erfc}\left[\frac{(k_x+1/2)\Delta K_x \ell_0}{2}\right]\right\}$$

$$\times \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{(k_y-1/2)\Delta K_y \ell_0}{2}\right] - \mathrm{erfc}\left[\frac{(k_y+1/2)\Delta K_y \ell_0}{2}\right]\right\} . \qquad (152)$$

The advantage of this approximation is that the error function terms depend only on the environment so they can be done once and used for all antennas thereby reducing the required processing time and array sizes.

A.3 RESULTS.

The error of the "exact" expression and the ACIRF approximation is calculated for a range of the ratio of the decorrelation distance to the antenna diameter. The scattering loss of the antenna is shown in Figure 7 for elevation pointing angles of 0, $\theta_0/2$, and θ_0. Figures 16, 17 and 18 show the relative error of the "exact" results and the ACIRF approximation for the same set of pointing angles. For these calculations a 32 by 32 angular grid is used (i.e., N_x and N_y are both set equal to 32). The ACIRF algorithm (dashed lines in the figures) has errors that approach those of the "exact" expression (bumpy solid lines in the figures) for large values of ℓ_0/D. In this case the antenna beamwidth is much larger than the angular spread of the scattering, and the beam profile is constant and equal to unity over the range of K values that contribute to the angular integral in Equation 140.

When the pointing angle is zero, the "exact" result in Figure 16 has a relatively constant error of about 4.7×10^{-5}. This error is due to inaccuracy in computing the complementary error function and round-off errors in the summation of the contributions from each grid cell. The ACIRF algorithm for this case has a peak error of about 1.1×10^{-3} that occurs when the angular spread of the incident power is about equal to the antenna beamwidth (i.e., ℓ_0 is approximately equal to D). Although this error is almost 25 times larger than the "exact" result error, it is small compared to one, and results in 0.1 percent error in the total power at the output of the antenna.

As the pointing angle increases, the maximum error of the "exact" result in Figures 17 and 18 remains small, less than 5×10^{-5}, because the angular grid is expanded in the direction the antenna is pointed. For non-zero pointing angles, the error changes sign over the plotted range of the ratio ℓ_0/D so, for some values of ℓ_0/D, the error is zero. The maximum error of the ACIRF algorithm can be several orders of magnitude larger than that of the "exact" expression, but again it is small compared to one, resulting in a maximum error of only 0.2 percent in the total power for these cases.

The ACIRF algorithm is used in channel modeling because it results in small errors in the total antenna output power and it requires significantly less computation than required by the "exact" expression when multiple antennas are modeled.

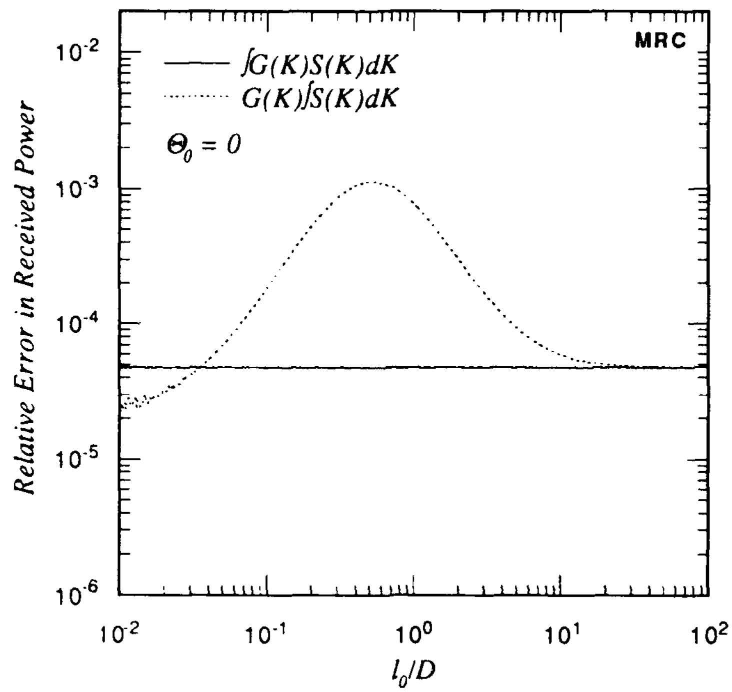

Figure 16. Relative error for pointing angle of 0.

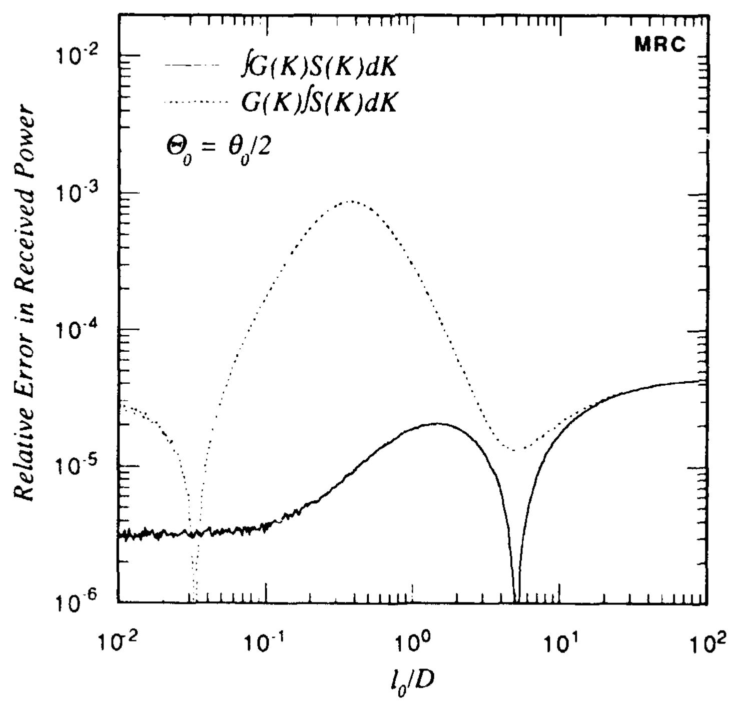

Figure 17. Relative error for pointing angle of $\theta_0/2$.

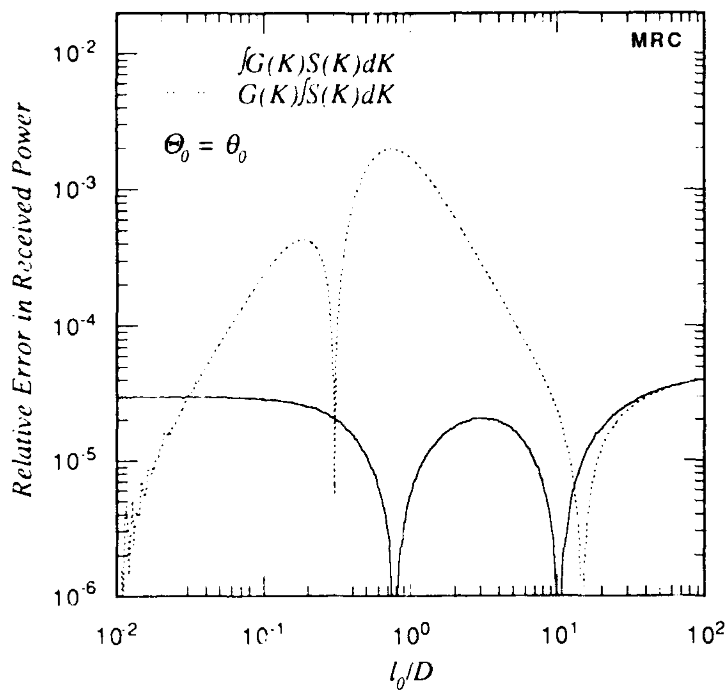

Figure 18. Relative error for pointing angle of θ_0.

APPENDIX B
ANGLE-DOPPLER GRID CELL POWER

This appendix describes the algorithm used to compute the angle-Doppler grid cell power, $E_{KD}(k_x,k_y,k_D)$, from the GPSD. This quantity is computed in the channel simulation on a grid that has a minimum of 32×32 angular cells and 150 Doppler cells. It is therefore necessary to have an efficient algorithm to compute the grid cell power to minimized the computation time of the channel simulation.

B.1 CALCULATION OF GRID CELL POWER.

The first section of this appendix is primarily a review of material presented in Section 3 of this report. Implementation details of the algorithms used to compute the angle-Doppler grid cell power are presented in the next section of this appendix.

B.1.1 Separation of Angular and Doppler Frequency Variables.

Recall from Section 3 that the most general form for $E_{KD}(k_x,k_y,k_D)$ is given by the expression:

$$E_{KD}(k_x,k_y,k_D) = \int_{(k_x-1/2)\Delta K_x}^{(k_x+1/2)\Delta K_x} \frac{dK_x}{2\pi} \int_{(k_y-1/2)\Delta K_y}^{(k_y+1/2)\Delta K_y} \frac{dK_y}{2\pi} \int_{(k_D-1/2)\Delta\omega_D}^{(k_D+1/2)\Delta\omega_D} \frac{d\omega_D}{2\pi} S_{KD}(\boldsymbol{K}_\perp,\omega_D) \ . \tag{153}$$

The integrand of this equation can be written in the form

$$S_{KD}(\boldsymbol{K}_\perp,\omega_D) = S_D(\omega_D)\, S_{KC}\left[K_x - \frac{C_{xt}\tau_0\omega_D}{\ell_x},\ K_y - \frac{C_{yt}\tau_0\omega_D}{\ell_y}\right] \tag{154}$$

where

$$S_D(\omega_D) = \sqrt{\pi}\,\tau_0 \exp\left[-\frac{\tau_0^2\omega_D^2}{4}\right] \tag{155}$$

and

$$S_{KC}(\boldsymbol{K}_\perp) = \frac{\pi\,\ell_x\ell_y}{\sqrt{1 - C_{xt}^2 - C_{yt}^2}} \times \exp\left[-\frac{K_x^2\ell_x^2(1 - C_{yt}^2) + K_y^2\ell_y^2(1 - C_{xt}^2) + 2C_{xt}C_{yt}K_x\ell_xK_y\ell_y}{4(1 - C_{xt}^2 - C_{yt}^2)}\right] . \tag{156}$$

As described in Section 3.2.3, two tricks are used to evaluate efficiently the grid cell power. The first trick is to take advantage of the translational properties of the GPSD. The power in a $K_x - K_y - \omega_D$ grid cell is

$$E_{KD}(k_x,k_y,k_D) = \int_{(k_D - 1/2)\Delta\omega_D}^{(k_D + 1/2)\Delta\omega_D} \frac{d\omega_D}{2\pi} S_D(\omega_D) \tag{157}$$

$$\times \int_{(k_x - 1/2)\Delta K_x}^{(k_x + 1/2)\Delta K_x} \frac{dK_x}{2\pi} \int_{(k_y - 1/2)\Delta K_y}^{(k_y + 1/2)\Delta K_y} \frac{dK_y}{2\pi} S_{KC}\left[K_x - \frac{C_{xt}\tau_0\omega_D}{\ell_x}, K_y - \frac{C_{yt}\tau_0\omega_D}{\ell_y}\right] .$$

The key to simplifying this expression is to note that the Doppler grid cell size is relatively small because of the large number of Doppler samples that are required to produce a long time realization. Thus it can be assumed that S_{KC} is constant over a Doppler cell, and Equation 157 reduces to a function of Doppler frequency times a shifted function of angle:

$$E_{KD}(k_x,k_y,k_D) = E_D(k_D)\, E_{KC}(k_x - m_x, k_y - m_y) \tag{158}$$

where

$$E_D(k_D) = \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{(k_D - 1/2)\tau_0\Delta\omega_D}{2}\right] - \mathrm{erfc}\left[\frac{(k_D + 1/2)\tau_0\Delta\omega_D}{2}\right]\right\} . \tag{159}$$

The quantity $E_{KC}(k_x - m_x, k_y - m_y)$ is the power in a shifted $K_x - K_y$ grid cell where

$$E_{KC}(k_x,k_y) = \int_{(k_x - 1/2)\Delta K_x}^{(k_x + 1/2)\Delta K_x} \frac{dK_x}{2\pi} \int_{(k_y - 1/2)\Delta K_y}^{(k_y + 1/2)\Delta K_y} \frac{dK_y}{2\pi} S_{KC}(\mathbf{K}_\perp) , \tag{160}$$

and the Doppler shift indices are given by

$$m_x = \mathrm{int}\left[\frac{C_{xt}\tau_0\omega_D}{\ell_x \Delta K_x}\right] \tag{161a}$$

$$m_y = \mathrm{int}\left[\frac{C_{yt}\tau_0\omega_D}{\ell_y \Delta K_y}\right] . \tag{161b}$$

Because of the $K_x - K_y$ cross terms in the expression for S_{KC}, an easily-evaluated closed-form result is still not obtainable for E_{KC}.

B.1.2 Evaluation of Angular Cell Power on a $K_p - K_q$ Grid.

The second trick used in the channel model technique is to note that a rotation by the angle ϑ (Eqn. 65) in the $K_x - K_y$ plane,

$$\vartheta = \frac{1}{2}\tan^{-1}\left[\frac{2C_{xt}C_{yt}\ell_x\ell_y}{\ell_x^2(1 - C_{xt}^2) - \ell_y^2(1 - C_{yt}^2)}\right] , \tag{162}$$

produces an orthogonal form of the GPSD that does not contain angular cross terms, and is therefore readily integrated. This orthogonalized GPSD is given by Equation 66 that has the following form for its angular part:

$$S_{KC}(K_p,K_q) = \frac{\pi\ell_p\ell_q}{\sqrt{(1 - C_{pt}^2)(1 - C_{qt}^2)}}\exp\left[-\frac{K_p^2\ell_p^2}{4(1 - C_{pt}^2)} - \frac{K_q^2\ell_q^2}{4(1 - C_{qt}^2)}\right] . \tag{163}$$

The quantities ℓ_p, ℓ_q, C_{pt}, and C_{qt} are given by Equations 67 and 68 in terms of the corresponding quantities defined in the x–y coordinate system. The signal power in a $K_p - K_q$ grid cell is

$$E_{KC}(k_p,k_q) = E_p(k_p)\,E_q(k_q) \tag{164}$$

where

$$E_p(k_p) = \frac{1}{2}\left\{\operatorname{erfc}\left[\frac{(k_p - 1/2)\Delta K_p\ell_p}{2\sqrt{1 - C_{pt}^2}}\right] - \operatorname{erfc}\left[\frac{(k_p + 1/2)\Delta K_p\ell_p}{2\sqrt{1 - C_{pt}^2}}\right]\right\} . \tag{165}$$

A similar expression holds for $E_q(k_q)$.

Now $E_{KC}(k_p,k_q)$ can be computed on a fine $K_p - K_q$ grid, and the values simply assigned to the $K_x - K_y$ grid cell in which they fall. The K_x - K_y cell indices are computed as follows:

$$k_x = \operatorname{int}\left[\frac{k_p\Delta K_p\cos\vartheta - k_q\Delta K_q\sin\vartheta}{\Delta K_x}\right] \tag{166a}$$

$$k_y = \operatorname{int}\left[\frac{k_p\Delta K_p\sin\vartheta + k_q\Delta K_q\cos\vartheta}{\Delta K_y}\right] . \tag{166b}$$

The total power in a $K_x - K_y$ grid cell is then the sum of all $E_{KC}(k_p,k_q)$ values that fall within the $K_x - K_y$ cell. Roughly ten $K_p - K_q$ grid cells are required within each K_x–K_y cell for this brute-force procedure to work. Thus the $K_p - K_q$ cell sizes are determined by the expressions:

$$\Delta K_p = \frac{0.1}{\left[\dfrac{\cos^2\vartheta}{(\Delta K_x)^2} + \dfrac{\sin^2\vartheta}{(\Delta K_y)^2}\right]^{\frac{1}{2}}} \tag{167a}$$

$$\Delta K_q = \frac{0.1}{\left[\dfrac{\sin^2\vartheta}{(\Delta K_x)^2} + \dfrac{\cos^2\vartheta}{(\Delta K_y)^2}\right]^{\frac{1}{2}}} . \tag{167b}$$

The size of the $K_p - K_q$ grid is also needed before $E_{KC}(k_p,k_q)$ can be computed. The one-dimensional form of $K_p - K_q$ angular power spectrum is

$$S_{KC}(K_p) = \frac{\sqrt{\pi}\,\ell_p}{\sqrt{1 - C_{pt}^2}} \exp\left[-\frac{K_p^2 \ell_p^2}{4(1 - C_{pt}^2)}\right] . \tag{168}$$

In order that the K_p grid contain a fraction ζ_0 of the angular power, the grid must extend to $K_{p,\max}$ where

$$\zeta_0 = \int_{-K_{p,max}}^{K_{p,max}} S_{KC}(K_p)\,\frac{dK_p}{2\pi} = \mathrm{erf}\left[\frac{K_{p,max}\,\ell_p}{2\sqrt{1 - C_{pt}^2}}\right] . \tag{169}$$

Solving for $K_{p,max}$ gives the result:

$$K_{p,max} = \kappa_{K,max}\,\frac{\sqrt{1 - C_{pt}^2}}{\ell_p} \tag{170a}$$

where $\kappa_{K,max}$ is given by Equation 103b. A similar expression holds for $K_{q,max}$:

$$K_{q,max} = \kappa_{K,max}\,\frac{\sqrt{1 - C_{qt}^2}}{\ell_q} . \tag{170b}$$

B.2 ALGORITHMS.

Implementation details for the evaluation of $E_{KC}(k_x - m_x, k_y - m_y)$ are discussed in this section. This implementation minimizes the number of computations of $E_{KC}(k_x - m_x, k_y - m_y)$ by using the shifting property for non-zero Doppler frequencies, and minimizes the number of computations of $E_{KC}(k_p,k_q)$ by carefully defining the region of the $K_p - K_q$ grid where $E_{KC}(k_x - m_x, k_y - m_y)$ is required.

B.2.1 Shifted Angular Grid Cell Power $E_{KC}(k_x - m_x, k_y - m_y)$.

In a computer implementation of the channel simulation, $E_{KC}(k_x,k_y)$ is an array with indices

$$k_x = -N_x/2,\ -N_x/2+1,\ \cdots,\ N_x/2-1 \tag{171a}$$

$$k_y = -N_y/2,\ -N_y/2+1,\ \cdots,\ N_y/2-1\ . \tag{171b}$$

The shifting process is then just a matter of rearranging the data within the array. Before discussing the algorithms used to shift the E_{KC} array, it is useful to understand the frequency at which this shifting process will occur.

Shifting Frequency. In evaluating the discrete impulse response function using Equation 97, the Doppler frequency discrete Fourier transform is performed last, after the two angular DFTs. Evaluation of the Doppler frequency spectral components $\hat{h}_A(j\Delta\tau, k_D\Delta\omega_D)$ starts at zero Doppler frequency ($k_D=0$) and proceeds to the maximum positive Doppler frequency ($k_D=N_D/2-1$). Spectral components for negative values of Doppler frequency can be obtained by taking advantage in the symmetry of the power in an angle-Doppler grid cell, $E_D(k_D)E_{KC}(k_x-m_x, k_y-m_y)$. Both E_D and E_{KC} are even functions of their arguments. Thus $E_D(-k_D)$ is equal to $E_D(k_D)$, and $E_{KC}[k_x+|m_x|, k_y+|m_y|]$ is equal to $E_{KC}[-k_x-|m_x|, -k_y-|m_y|]$. Therefore the Doppler frequency spectral components for negative Doppler frequencies can be evaluated using the angle-Doppler grid cell power calculated for the corresponding positive Doppler frequencies with $k_x\Delta K_x$ and $k_y\Delta K_y$ replaced by $-k_x\Delta K_x$ and $-k_y\Delta K_y$ in Equation 97. (The residual Doppler shifts, ε_x and ε_y, in Equation 97 also change signs for negative Doppler frequencies.)

Now as each new Doppler frequency spectral component is computed (i.e., for each value of k_D, $k_D=1, 2, \cdots, N_D/2-1$), the incremental Doppler shift indices are

$$m_x = \text{int}\left[\frac{C_{xt}\tau_0 k_D\Delta\omega_D}{\ell_x\Delta K_x} - M_x(k_D-1)\right] \tag{172a}$$

$$m_y = \text{int}\left[\frac{C_{yt}\tau_0 k_D\Delta\omega_D}{\ell_y\Delta K_y} - M_y(k_D-1)\right] \tag{172b}$$

where $M_x(k_D)$ and $M_y(k_D)$ are the cumulative shift indices:

$$M_x(k_D) = \sum_{m_D=0}^{k_D} m_x(m_D)\ ,\ M_x(-1) = 0 \tag{173a}$$

$$M_y(k_D) = \sum_{m_D=0}^{k_D} m_y(m_D)\ ,\ M_y(-1) = 0\ . \tag{173b}$$

The corresponding residual shifts are:

$$\varepsilon_x = \frac{C_{xt}\tau_0 k_D \Delta\omega_D}{\ell_x} - M_x(k_D)\Delta K_x \tag{174a}$$

$$\varepsilon_y = \frac{C_{yt}\tau_0 k_D \Delta\omega_D}{\ell_y} - M_y(k_D)\Delta K_x \ . \tag{174b}$$

The normalized Doppler frequency grid cell size ($\tau_0\Delta\omega_D$) is small compared to the normalized angular grid cell sizes ($\ell_x\Delta K_x$ and $\ell_y\Delta K_y$) because there are generally more Doppler grid cells than angular grid cells (in any one dimension). Thus the incremental Doppler shift indices m_x and m_y may be zero for several sequential values of k_D. Shifting of the $E_{KC}(k_x,k_y)$ array in the x-direction is necessary approximately every $\ell_x\Delta K_x/C_{xt}\tau_0\Delta\omega_D$ Doppler cells, and shifting in the y-direction is necessary approximately every $\ell_y\Delta K_y/C_{yt}\tau_0\Delta\omega_D$ Doppler cells. Note, however, that the residual shifts will change for every new value of k_D.

Shifting Algorithm. Assume for the moment that $E_{KC}(k_x,k_y)$ has been computed and that the shifted array $E_{KC}(k_x-m_x,k_y-m_y)$ is desired. The actual computation of $E_{KC}(k_x,k_y)$ for arbitrary k_x and k_y will be discussed in the next subsection.

For non-negative values of C_{xt} and C_{yt}, the incremental Doppler shift indices m_x and m_y will also be non-negative. However, the general model puts no restrictions on the signs of the space-time correlation coefficients as long as the square root of the sum of the squares of these coefficients is between zero and one. Therefore, a completely general algorithm must include the possibility of positive and negative values of the incremental Doppler shifts.

Now assume that $E_{KC}(k_x-m_x,k_y)$ is desired where m_x is positive. An algorithm that performs this shifting is

$$\begin{array}{lcl} E_{KC}(N_x/2-1,k_y) & \leftarrow & E_{KC}(N_x/2-1-m_x,k_y) \\ E_{KC}(N_x/2-2,k_y) & \leftarrow & E_{KC}(N_x/2-2-m_x,k_y) \\ & \bullet & \\ & \bullet & \\ & \bullet & \\ E_{KC}(-N_x/2+m_x,k_y) & \leftarrow & E_{KC}(-N_x/2,k_y) \ . \end{array} \tag{175}$$

Note that after the shifting, $E_{KC}(-N_x/2,k_y)$ through $E_{KC}(-N_x/2+m_x-1,k_y)$ have not been defined. These grid cell powers will then need to be computed after the shifting is performed.

If m_x is negative, a shifting algorithm is

$$
\begin{aligned}
E_{KC}(-N_x/2,k_y) &\leftarrow E_{KC}(-N_x/2-m_x,k_y) \\
E_{KC}(-N_x/2+1,k_y) &\leftarrow E_{KC}(-N_x/2+1-m_x,k_y) \\
&\vdots \\
E_{KC}(N_x/2-1+m_x,k_y) &\leftarrow E_{KC}(N_x/2-1,k_y) \ .
\end{aligned}
\tag{176}
$$

Note that after the shifting, $E_{KC}(N_x/2+m_x,k_y)$ through $E_{KC}(N_x/2-1,k_y)$ have not been defined and will need to be computed. Similar algorithms can be defined to shift $E_{KC}(N_x,k_y+m_y)$ by positive or negative m_y. The algorithm for computing $E_{KC}(k_x,k_y)$ is discussed next.

B.2.2 Angular Grid Cell Power $E_{KC}(k_x,k_y)$.

Depending on the Doppler frequency, E_{KC} may be computed over all or part of the K_x-K_y grid. When the Doppler frequency is zero and E_{KC} is computed for the first time, then E_{KC} is computed over the entire angular grid. However, for positive values of the Doppler frequency, E_{KC} is obtained by shifting and new values of E_{KC} are required only in a small region of the K_x-K_y grid. Angular grid cell power values for negative Doppler frequencies are obtained directly from the corresponding positive Doppler frequency values, and no new calculations of E_{KC} are required.

This section describes an algorithm for computing $E_{KC}(k_x,k_y)$ with arbitrary limits on the indices. In general the limits on k_x and k_y are $k_{x,1}$ to $k_{x,2}$ and $k_{y,1}$ to $k_{y,2}$. When E_{KC} is computed for the first time,

$$k_{x,1} = -N_x/2 \tag{177a}$$

$$k_{x,2} = N_x/2-1 \tag{177b}$$

$$k_{y,1} = -N_y/2 \tag{177c}$$

$$k_{y,2} = N_y/2-1 \ , \tag{177d}$$

and afterward,

$$k_{x,1} = \begin{cases} -N_x/2 & \text{if } m_x > 0 \\ N_x/2 + m_x & \text{if } m_x < 0 \end{cases} \tag{178a}$$

$$k_{x,2} = \begin{cases} -N_x/2 + m_x - 1 & \text{if } m_x > 0 \\ N_x/2 - 1 & \text{if } m_x < 0 \end{cases} \tag{178b}$$

$$k_{y,1} = \begin{cases} -N_y/2 & \text{if } m_y > 0 \\ N_y/2 + m_y & \text{if } m_y < 0 \end{cases} \quad (178c)$$

$$k_{y,2} = \begin{cases} -N_y/2 + m_y - 1 & \text{if } m_y > 0 \\ N_y/2 - 1 & \text{if } m_y < 0 \end{cases} \quad (178d)$$

$K_p - K_q$ Regions. Given these limits, the first task is to compute the $K_x - K_y$ region on the $K_p - K_q$ grid defined by the limits. This rectangular region is defined by the four points $(K_{x,1}, K_{y,1})$, $(K_{x,1}, K_{y,2})$, $(K_{x,2}, K_{y,1})$, and $(K_{x,2}, K_{y,2})$ where

$$K_{x,1} = \left[k_{x,1} - \frac{1}{2} - M_x(k_D)\right] \Delta K_x \quad (179a)$$

$$K_{x,2} = \left[k_{x,1} + \frac{1}{2} - M_x(k_D)\right] \Delta K_x \quad (179b)$$

$$K_{y,1} = \left[k_{y,1} - \frac{1}{2} - M_y(k_D)\right] \Delta K_y \quad (179c)$$

$$K_{y,2} = \left[k_{y,1} + \frac{1}{2} - M_y(k_D)\right] \Delta K_y \ . \quad (179d)$$

The corresponding $K_p - K_q$ coordinates are

$$K_{p,1} = K_{x,1} \cos \vartheta + K_{y,1} \sin \vartheta \quad (180a)$$

$$K_{p,2} = K_{x,1} \cos \vartheta + K_{y,2} \sin \vartheta \quad (180b)$$

$$K_{p,3} = K_{x,2} \cos \vartheta + K_{y,2} \sin \vartheta \quad (180c)$$

$$K_{p,4} = K_{x,2} \cos \vartheta + K_{y,1} \sin \vartheta \quad (180d)$$

$$K_{q,1} = K_{y,1} \cos \vartheta - K_{x,1} \sin \vartheta \quad (180e)$$

$$K_{q,2} = K_{y,2} \cos \vartheta - K_{x,1} \sin \vartheta \quad (180f)$$

$$K_{q,3} = K_{y,2} \cos \vartheta - K_{x,2} \sin \vartheta \quad (180g)$$

$$K_{q,4} = K_{y,1} \cos \vartheta - K_{x,2} \sin \vartheta \ . \quad (180h)$$

The algorithm that determines the region of the $K_p - K_q$ plane that is encompassed by the $K_x - K_y$ region and that contains signal energy depends on $K_{p,1}$ being the smallest K_p value. If, in fact, $K_{p,2}$ is less than $K_{p,1}$, then the $K_p - K_q$ coordinates must be renamed:

$$K_p = K_{p,1} \tag{181a}$$

$$K_q = K_{q,1} \tag{181b}$$

$$K_{p,1} = K_{p,2} \tag{181c}$$

$$K_{q,1} = K_{q,2} \tag{181d}$$

$$K_{p,2} = K_{p,3} \tag{181e}$$

$$K_{q,2} = K_{q,3} \tag{181f}$$

$$K_{p,3} = K_{p,4} \tag{181g}$$

$$K_{q,3} = K_{q,4} \tag{181h}$$

$$K_{p,4} = K_p \tag{181i}$$

$$K_{q,4} = K_q \ . \tag{181j}$$

With this ordering of the $K_p - K_q$ coordinates, the smallest value of K_p is $K_{p,1}$ and the largest value is $K_{p,3}$. The smallest value of K_q is $K_{q,4}$ and the largest value is $K_{q,2}$. Thus the $K_x - K_y$ region is outside the limits of the $K_p - K_q$ grid if $K_{p,3} < -K_{p,max}$, $K_{p,1} > K_{p,max}$, $K_{q,2} < K_{q,max}$, or $K_{q,4} > K_{q,max}$. If none of these conditions are met, then there is signal energy within the $K_x - K_y$ region, and the calculation continues. If any of these conditions are met, then the $K_x - K_y$ region falls outside the region in the $K_p - K_q$ plane where there is signal power, and there is no need to continue the calculation.

There are nine separate cases, illustrated in Figure 19, when the overlap between the $K_x - K_y$ region and the $K_p - K_q$ region containing 99.9 percent of the signal energy is considered. The rectangle for each case corresponds to the $K_x - K_y$ region over which the calculation of E_{KC} is to be done. The numbered corners correspond to the $K_p - K_q$ coordinates of the $K_x - K_y$ rectangle given by Equation 180. The dashed lines illustrate the $\pm K_{q,max}$ limits of the $K_p - K_q$ grid. Case 1 configurations are determined by the condition that $-K_{q,max} < K_{q,1} < K_{q,max}$. Case 2 configurations occur when this condition is not met. The shaded areas in the figures illustrate the part of the $K_p - K_q$ plane bounded by the $K_x - K_y$ region that also meet the condition $-K_{q,max} < K_q < K_{q,max}$.

$K_p - K_q$ Power Centroid Lines. Once the $K_x - K_y$ region on the $K_p - K_q$ plane has been defined, an efficient method of computing the signal power is to draw a power centroid line across the region, as illustrated by the lines across the shaded areas in Figure 19. In two cases (2a-1 and 2b-1), this line is colinear with the $\pm K_{q,max}$ line.

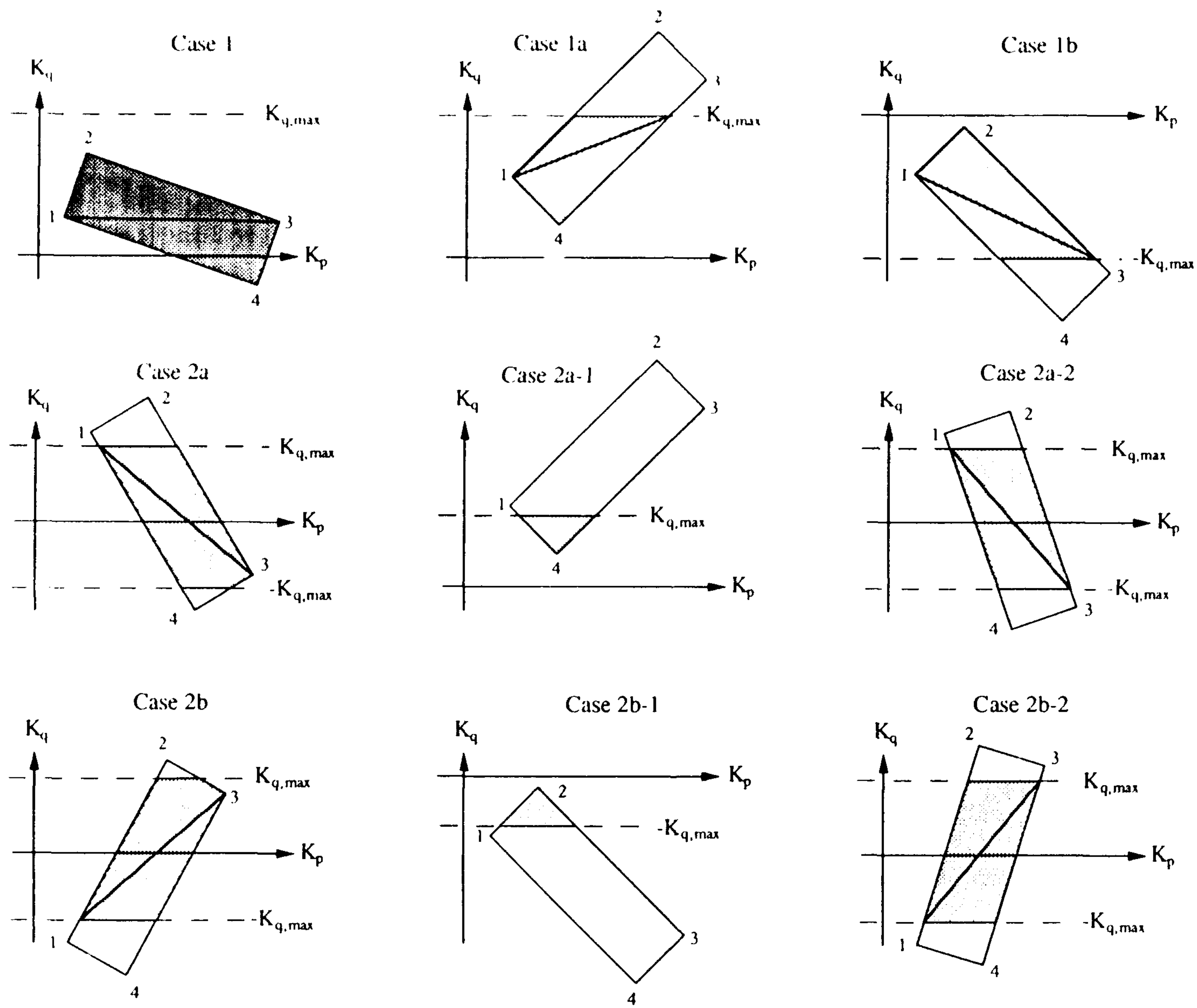

Figure 19. $K_x - K_y$ **regions on the** $K_p - K_q$ **plane.**

The power in $K_p - K_q$ grid cells is calculated for each value of K_p in the shaded regions, by starting with the cell on the line and proceeding to larger values of K_q until the upper K_q boundary of the shaded region is encountered. Then the power in the first cell below the line is calculated and so on until the lower K_q boundary of the shaded region is encountered.

The endpoints of the line depend on the case. For case 1:

$$K_{p,start} = K_{p,1} \tag{182a}$$

$$K_{p,stop} = K_{p,3} \tag{182b}$$

$$K_{q,start} = K_{q,1} \tag{182c}$$

$$K_{q,stop} = K_{q,3} \ . \tag{182d}$$

For case 1a:

$$K_{p,start} = K_{p,1} \tag{183a}$$

$$K_{p,stop} = K_{p,4} + (K_{q,max} - K_{q,4}) \frac{K_{p,3} - K_{p,4}}{K_{q,3} - K_{q,4}} \tag{183b}$$

$$K_{q,start} = K_{q,1} \tag{183c}$$

$$K_{q,stop} = K_{q,max} \ . \tag{183d}$$

For case 1b:

$$K_{p,start} = K_{p,1} \tag{184a}$$

$$K_{p,stop} = K_{p,2} - (K_{q,max} + K_{q,2}) \frac{K_{p,3} - K_{p,2}}{K_{q,3} - K_{q,2}} \tag{184b}$$

$$K_{q,start} = K_{q,1} \tag{184c}$$

$$K_{q,stop} = -K_{q,max} \ . \tag{184d}$$

For case 2a:

$$K_{p,start} = K_{p,1} + (K_{q,max} - K_{q,1}) \frac{K_{p,4} - K_{p,1}}{K_{q,4} - K_{q,1}} \tag{185a}$$

$$K_{p,stop} = K_{p,3} \tag{185b}$$

$$K_{q,start} = K_{q,max} \tag{185c}$$

$$K_{q,stop} = K_{q,3} \ . \tag{185d}$$

For case 2a-1:

$$K_{p,start} = K_{p,1} + (K_{q,max} - K_{q,1}) \frac{K_{p,4} - K_{p,1}}{K_{q,4} - K_{q,1}} \tag{186a}$$

$$K_{p,stop} = K_{p,4} + (K_{q,max} - K_{q,4}) \frac{K_{p,3} - K_{p,4}}{K_{q,3} - K_{q,4}} \tag{186b}$$

$$K_{q,start} = K_{q,max} \tag{186c}$$

$$K_{q,stop} = K_{q,max} \ . \tag{186d}$$

For case 2a-2:

$$K_{p,start} = K_{p,1} + (K_{q,max} - K_{q,1}) \frac{K_{p,4} - K_{p,1}}{K_{q,4} - K_{q,1}} \tag{187a}$$

$$K_{p,stop} = K_{p,2} - (K_{q,max} + K_{q,2}) \frac{K_{p,3} - K_{p,2}}{K_{q,3} - K_{q,2}} \tag{187b}$$

$$K_{q,start} = K_{q,max} \tag{187c}$$

$$K_{q,stop} = -K_{q,max} \ . \tag{187d}$$

For case 2b:

$$K_{p,start} = K_{p,1} - (K_{q,max} + K_{q,1}) \frac{K_{p,2} - K_{p,1}}{K_{q,2} - K_{q,1}} \tag{188a}$$

$$K_{p,stop} = K_{p,3} \tag{188b}$$

$$K_{q,start} = -K_{q,max} \tag{188c}$$

$$K_{q,stop} = K_{q,3} \ . \tag{188d}$$

For case 2b-1:

$$K_{p,start} = K_{p,1} - (K_{q,max} + K_{q,1}) \frac{K_{p,2} - K_{p,1}}{K_{q,2} - K_{q,1}} \tag{189a}$$

$$K_{p,stop} = K_{p,3} - (K_{q,max} + K_{q,3}) \frac{K_{p,2} - K_{p,3}}{K_{q,2} - K_{q,3}} \tag{189b}$$

$$K_{q,start} = -K_{q,max} \tag{189c}$$

$$K_{q,stop} = -K_{q,max} \ . \tag{189d}$$

Finally, for case 2b-2:

$$K_{p,start} = K_{p,1} - (K_{q,max} + K_{q,1}) \frac{K_{p,2} - K_{p,1}}{K_{q,2} - K_{q,1}} \quad (190a)$$

$$K_{p,stop} = K_{p,4} + (K_{q,max} - K_{q,4}) \frac{K_{p,3} - K_{p,4}}{K_{q,3} - K_{q,4}} \quad (190b)$$

$$K_{q,start} = -K_{q,max} \quad (190c)$$

$$K_{q,stop} = K_{q,max} \ . \quad (190d)$$

Once the endpoints of the line are defined, the slope of the line is

$$slope = \frac{K_{q,stop} - K_{q,\mathrm{start}}}{K_{p,stop} - K_{p,\mathrm{start}}} \ . \quad (191)$$

$K_p - K_q$ and $K_x - K_y$ Grid Power. The final step is to compute the $K_p - K_q$ grid cell power and to assign that power to $K_x - K_y$ grid cells within the $K_x - K_y$ region.

The indices of the K_p grid cells within the $K_x - K_y$ region are

$$k_{p,start} = \mathrm{int}\left[\frac{\max(K_{p,start}, -K_{p,max})}{\Delta K_p} + \frac{1}{2}\,\mathrm{sign}\,(K_{p,start})\right] \quad (192a)$$

$$k_{p,stop} = \mathrm{int}\left[\frac{\min(K_{p,stop}, K_{p,max})}{\Delta K_p} + \frac{1}{2}\,\mathrm{sign}\,(K_{p,stop})\right] , \quad (192b)$$

where sign $(\cdot)$ is the sign function (i.e., a function that is equal to +1 if the argument is positive and is equal to −1 otherwise) and the maximum k_q index is

$$k_{q,max} = \mathrm{int}\left[\frac{K_{q,max}}{\Delta K_q} + \frac{1}{2}\,\mathrm{sign}\,(K_{q,max})\right] \ . \quad (193)$$

The minimum and maximum functions that appear in Equation 192 constrain $K_x - K_y$ region to be within the $\pm K_{p,max}$ bounds where 99.9 percent of the signal energy lies. The 1/2 sign (K) terms cause the integer function to round its argument in the desired way.

Now a loop is executed over the k_p index, starting at $k_{p,start}$ and ending at $k_{p,stop}$. For each value of k_p, the energy in a ΔK_p grid cell is given by Equation 165 that is reproduced here:

$$E_p(k_p) = \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{(k_p - 1/2)\Delta K_p \ell_p}{2\sqrt{1 - C_{pt}^2}}\right] - \mathrm{erfc}\left[\frac{(k_p + 1/2)\Delta K_p \ell_p}{2\sqrt{1 - C_{pt}^2}}\right]\right\} \quad (165)$$

Corresponding to each value of k_p is a range of k_q values. The loop over K_q values starts at the power centroid line, which has a K_q value of

$$K_{q,L} = K_{q,start} + (k_p \Delta K_p - K_{p,start}) \frac{K_{q,stop} - K_{q,\mathrm{start}}}{K_{p,stop} - K_{p,\mathrm{start}}} \tag{194}$$

and an index of

$$k_{q,L} = \mathrm{int}\left[\frac{K_{q,L}}{\Delta K_q} + \frac{1}{2}\,\mathrm{sign}\,(K_{q,L})\right] . \tag{195}$$

The K_q loop starts at the k_q value of the line and proceeds to higher values of k_q until the limits of the $K_x - K_y$ region are encountered, as described below. Then the calculation is restarted at the first K_q cell below the line and k_q is decremented until the $K_x - K_y$ region boundary is again reached. For each K_q grid cell, the signal power is

$$E_q(k_q) = \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{(k_q - 1/2)\Delta K_q \ell_q}{2\sqrt{1 - C_{qt}^2}}\right] - \mathrm{erfc}\left[\frac{(k_q + 1/2)\Delta K_q \ell_q}{2\sqrt{1 - C_{qt}^2}}\right]\right\} \tag{196}$$

and the total $K_p - K_q$ grid cell power is just $E_p(k_p)E_q(k_q)$.

Finally the $K_p - K_q$ grid cell power is assigned to $K_x - K_y$ grid cells, and $E_p(k_p)E_q(k_q)$ is added to the power already assigned to each $K_x - K_y$ grid cell. The $K_x - K_y$ grid cell indices are computed as

$$k_x = \mathrm{int}\left[\frac{k_p \Delta K_p \cos\vartheta - k_q \Delta K_q \sin\vartheta + M_x(k_D)\Delta K_x}{\Delta K_x}\right] \tag{197a}$$

$$k_y = \mathrm{int}\left[\frac{k_p \Delta K_p \sin\vartheta + k_q \Delta K_q \cos\vartheta + M_y(k_D)\Delta K_y}{\Delta K_y}\right] . \tag{197b}$$

These indices are also used to stop the loop over k_q. This loop is terminated whenever k_x or k_y fall outside the limits given by Equations 177 or 178. Thus when k_q is being incremented for cells above the power centroid line, the k_q loop is continued as long as $k_{x,1} \le k_x \le k_{x,2}$ and $k_{y,1} \le k_y \le k_{y,2}$. When either of these conditions are not met, the loop is reset to the first k_q value below the line and k_q is decremented as long as $k_{x,1} \le k_x \le k_{x,2}$ and $k_{y,1} \le k_y \le k_{y,2}$. When either of these conditions are not met, the next value in the k_p loop is executed.

APPENDIX C
SAMPLED RAYLEIGH FADING

This appendix is intended to address the question: How close is an ACIRF realization of the impulse response function to Rayleigh fading? Closely related questions that arise during simulation or hardware testing activities using ACIRF realizations are: How many decorrelation times per realization are necessary? How many samples per decorrelation time are necessary? How should interpolation be done between samples? These questions are answered in part in the DNA signal specification for nuclear scintillation [*Wittwer*, 1980] which requires a minimum of 100 decorrelation times per realization and 10 samples per decorrelation time. However, experience has shown that receiver performance can show considerable statistical variation when the minimum realization length is used. This is particularly true of links that have large power margins and are susceptible to only the deepest fades. Of course the best way to answer these questions is to measure link performance with realizations of increasing length and resolution until the statistical variation in the results from one realization to the next is acceptable. Unfortunately, the luxury of doing this analysis often does not exist.

The next higher level of analysis of these questions is to look at the statistics of ACIRF realizations. This is the approach that will be taken in this appendix. The first-order statistics of the realizations are measured by calculating the cumulative distribution of the amplitudes and amplitude moments and comparing these quantities with ensemble values for Rayleigh fading. The second-order statistics of the realizations are measured by calculating the mean duration and separation of fades and again comparing these quantities to ensemble values.

In general, the received signal $r(t)$ may be written as the convolution of the channel impulse response function $h(\tau,t)$ with the transmitted modulation $m(t)$ that is given by Equation (1):

$$r(t) = \int_0^\infty h(\tau,t)\, m(t-\tau)\, d\tau \ . \tag{1}$$

In either software link simulations or in hardware channel simulators, Equation (1) can be implemented as a tapped delay line:

$$r(t) = \sum_{j=0}^{N_\tau - 1} h(j\Delta\tau,t)\, m(t-j\Delta\tau)\, \Delta\tau \tag{198}$$

where N_τ is number of taps on the delay line; $\Delta\tau$ is the delay spacing of the delay line; $h(t,j\Delta\tau)$ is the time varying complex weight of the j^{th} tap; and $m(t)$ is the input signal modulation. In a software simulation of link performance, time may also be discretely sampled (i.e., $t = k_T\Delta t$).

Under Rayleigh fading conditions, $h(\tau,t)$ is a complex, zero mean, normally distributed random variable and thus has a Rayleigh amplitude distribution. It then follows from Equation (170) that $r(t)$ is also a complex, zero mean, normally distributed random variable with a Rayleigh amplitude distribution.

A complete analysis of these issues would consider the statistics of each delay of the discrete impulse response function $h(j\Delta\tau,k_T\Delta t)$. However, this is beyond the scope of this appendix. Therefore, statistical measurements of the flat fading impulse response function $h(k_T\Delta t)$, where

$$h(k_T\Delta t) = \sum_{j=0}^{N_\tau - 1} h(j\Delta\tau,k_T\Delta t)\Delta\tau \ , \tag{199}$$

will be presented in this appendix. The statistics of $h(k_T\Delta t)$ will give some indication of the statistics of the frequency selective impulse response function $h(j\Delta\tau,k_T\Delta t)$.

Although the realizations of the impulse response function are usually generated with samples spaced at $\tau_0/10$, the sampling period of simulated or tested receivers can be much smaller than $\tau_0/10$. There are at least three approaches to this problem. One approach is to sample the impulse response function at a rate equal to the sample rate of the receiver. However, if the sample period is much smaller than τ_0, this results in very large realizations in order for each realization have the required *100* decorrelation times. Another approach is to hold the impulse response function constant during the period $\tau_0/10$, and change it abruptly at the end of the period to the next value of the impulse response function. In principle this procedure is acceptable because $\tau_0/10$ sampling should result in small changes from one value of the impulse response function to the next. A third approach is to interpolate between values of the impulse response function.

For a finite length realization, measured statistics will be random variables with some mean and standard deviation. The variation of the measured values about the ensemble values should go approximately as $(N_0/N_T)^{1/2}$ where N_T/N_0 is the number of decorrelation times in the realization (i.e., the number of "independent samples" of Rayleigh fading). To show the variation of measured statistics of the impulse response function generated by ACIRF, 1000 independent realizations were created with $N_T = 1024$, 2048, or 4096 time samples and $N_0 = 10$ samples per decorrelation time. Measurements of first- and second-order statistics are compared with ensemble values in the next subsections.

The time between realization points is $\Delta t = \tau_0/N_0$. *Dana* [1988] has shown that $N_0 = 10$ is sufficient when the sample interval T_S of the realizations is

$$T_S = \Delta t/4 = \tau_0/40 \tag{200}$$

using linear interpolation. This sampling has been used to generate the results below.

C.1 FIRST-ORDER STATISTICS.

One criterion for deciding that a realization has Rayleigh amplitude statistics is that the moments of the amplitude should agree with Rayleigh values. Ensemble values for the moments of the amplitude are easily obtained from the probability density function for Rayleigh fading:

$$f(a) = \frac{2a}{P_0} \exp\left[-\frac{a^2}{P_0}\right] \tag{201}$$

where a is the amplitude of the fading signal [i.e., $a(t) = \sqrt{r(t)r^*(t)}$] and P_0 is the mean power of the realization. The n^{th} moment of the amplitude is computed using the equation

$$\langle a^n \rangle = \int_0^\infty a^n f(a)\, da \ . \tag{202}$$

The first four moments of the amplitude and the scintillation index S_4 are:

$$\langle a \rangle = \frac{1}{2}\sqrt{\pi P_0} \tag{203a}$$

$$\langle a^2 \rangle = P_0 \tag{203b}$$

$$\langle a^3 \rangle = \frac{3}{4}\sqrt{\pi P_0^3} \tag{203c}$$

$$\langle a^4 \rangle = 2P_0^2 \tag{203d}$$

$$S_4 = \left[\frac{\langle a^4 \rangle - \langle a^2 \rangle^2}{\langle a^2 \rangle^2}\right]^{\frac{1}{2}} \ . \tag{203e}$$

It is necessary, but not sufficient, that S_4 equal unity for Rayleigh fading. The scintillation index is a good measure of the statistics of flares but not of fades.

Statistics that are sensitive to the distribution of fades are the first two moments of the log amplitude, $\langle \chi \rangle$ and $\langle \chi^2 \rangle$:

$$\langle \chi \rangle = \langle \ln a \rangle = \frac{1}{2} \ln P_0 - \frac{\gamma}{2} \tag{204a}$$

$$\langle \chi^2 \rangle = \langle \ln^2 a \rangle = \langle \chi \rangle^2 + \frac{\pi^2}{24} \tag{204b}$$

where γ is Euler's constant ($\gamma = 0.5772157\cdots$).

To measure the expected variation of the first-order statistics of a realization, amplitude moments were measured for each realization. For example, the measured first moment of the amplitude of the k^{th} realization $h_k(k_T \Delta t)$ is given by

$$\mu_k = \frac{1}{N_T} \sum_{k_T = 1}^{N_T} \sqrt{h_k(k_T \Delta t) h_k^*(k_T \Delta t)} \ . \tag{205}$$

The mean and standard deviation of μ_k is then computed from the values for the 1000 realizations. Similar equations hold for the other moments.

Measured values of the mean and standard deviation (denoted sigma) of the amplitude moments, S_4, and the first two moments of the log amplitude are presented in Table 11 for N_T equal to 1024, 2048, and 4096 and N_0 equal to 10 in all cases. Measured values in the table have been normalized to their ensemble values.

Measured values for a single realization should equal the average values plus or minus one or two standard deviations. It can be seen from the table that the average values are close to the ensemble values (i.e., the normalized mean values are close to unity) but the standard deviations of the higher moments can be as large as 20 percent of the average values for the shorter $100\tau_0$ (N_T = 1024, N_0 = 10) realizations. The data also show that the standard deviations do indeed go as $(N_0/N_T)^{1/2}$ because the values for N_T = 4096 realizations are roughly half the corresponding values for N_T = 1024 realizations.

Table 11. Amplitude moments of ACIRF realizations.

	N_T = 1024		N_T = 2048		N_T = 4096	
Moment	Mean*	Sigma*	Mean*	Sigma*	Mean*	Sigma*
$\langle a \rangle$	0.9993	0.0544	0.9983	0.0398	0.9988	0.0291
$\langle a^2 \rangle$	0.9976	0.1053	0.9959	0.0772	0.9978	0.0559
$\langle a^3 \rangle$	0.9949	0.1607	0.9927	0.1173	0.9967	0.0849
$\langle a^4 \rangle$	0.9914	0.2258	0.9885	0.1636	0.9954	0.1191
S_4	0.9820	0.0844	0.9891	0.0610	0.9958	0.0460
$\langle \chi \rangle$	0.9861	0.1063	0.9924	0.0775	0.9973	0.0578
$\langle \chi^2 \rangle$	0.9859	0.1379	0.9930	0.1010	0.9969	0.0739

*Mean and sigma normalized to ensemble values

Another criterion for the first order statistics is close agreement between the measured cumulative distributions of the realizations and the Rayleigh cumulative distribution $F(P)$:

$$F(P) = \int_0^{\sqrt{P}} f(a)\, da = 1 - \exp\left[-\frac{P}{P_0}\right] \tag{206}$$

where P is instantaneous power ($P = a^2$). Measured cumulative distributions (dots plus or minus one-sigma error bars) are plotted in Figures 20a, 20b, and 20c for the three values of N_T along with the ensemble curve (Eqn. 206). One sigma error bars are computed by measuring the variation of the cumulative distribution points over the 1000 realizations. Mean values for the 1000 realizations are plotted as solid dots in the figures.

It can be seen from Figure 20a that $100\tau_0$ realizations do indeed have, on the average, a Rayleigh distribution of fades down to at least 30 dB. It is clear, however, that the possible deviation from Rayleigh fading of a single realization becomes larger as one examines deeper fades. For larger values of N_T, as shown in Figures 20b and 20c, the variation of the cumulative distribution for a single realization from Rayleigh fading becomes less, again decreasing roughly as $(N_0/N_T)^{1/2}$.

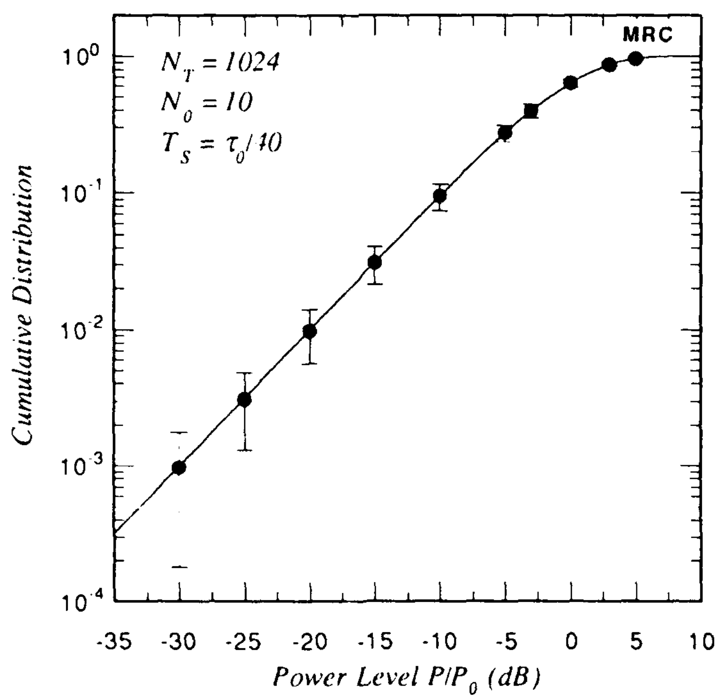

Figure 20a. Cumulative distribution of $100\tau_0$ realizations.

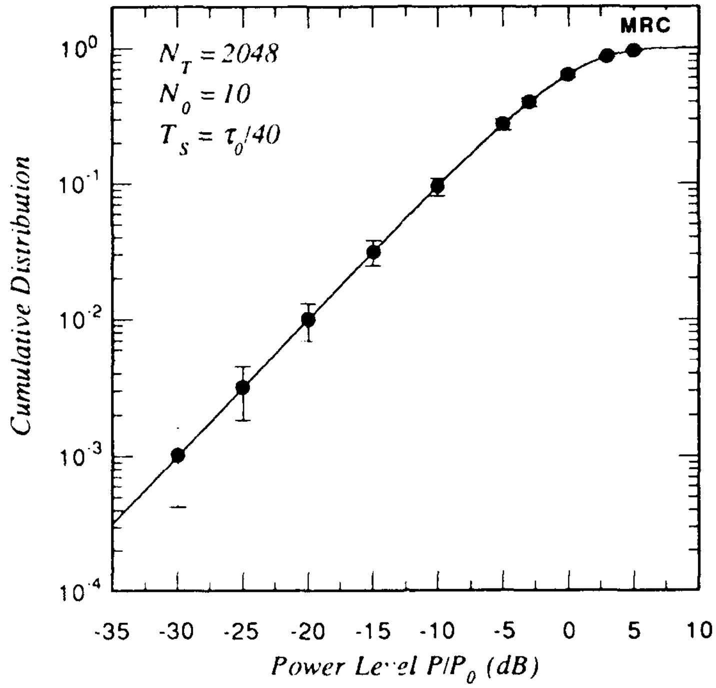

Figure 20b. Cumulative distribution of $200\tau_0$ realizations.

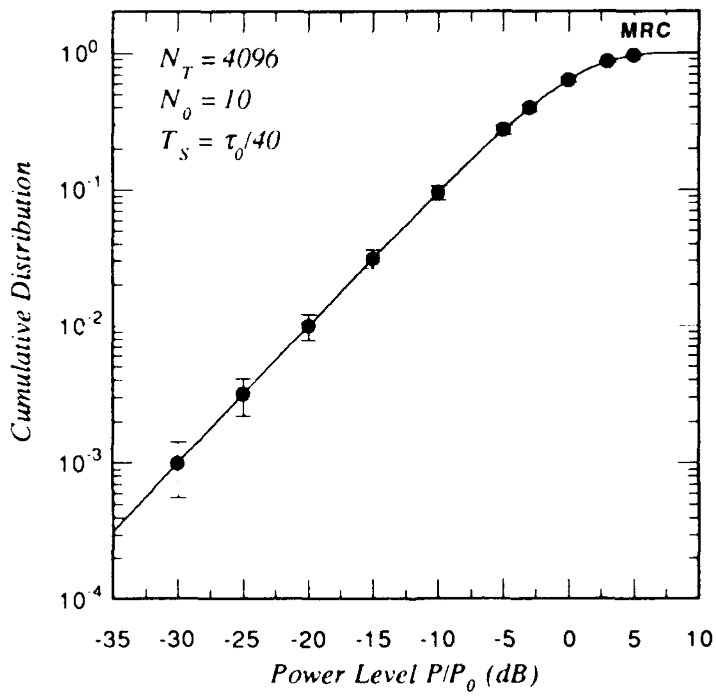

Figure 20c. Cumulative distribution of $400\tau_0$ realizations.

C.2 SECOND-ORDER STATISTICS.

The fidelity of ACIRF realizations in reproducing the second order statistics of the fading will be demonstrated by comparing the mean fade duration and separation with ensemble values. Ensemble values for the mean duration T_{dur} and separation T_{sep} of fades below the power level P are [*Dana*, 1988]:

$$\frac{\langle T_{dur}(P)\rangle}{\tau_0} = \left[\frac{\pi P_0}{2P}\right]^{\frac{1}{2}} \left\{\exp\left[\frac{P}{P_0}\right] - 1\right\} \tag{207a}$$

$$\frac{\langle T_{sep}(P)\rangle}{\tau_0} = \left[\frac{\pi P_0}{2P}\right]^{\frac{1}{2}} \exp\left[\frac{P}{P_0}\right] \tag{207b}$$

The mean fade duration is a good statistic to examine for communications applications because errors often occur in bursts during deep fades. If the fades, on the average, are too long or too short, the error bursts will not have the proper durations and the resulting receiver performance may be misleading. Fade duration measurements and the ensemble curve for a Gaussian Doppler spectrum are shown in Figures 21a, 21b, and 21c for the three values of N_T.

The fact that the measured values mean of fade duration do not level off at τ_0/N_0 is due to sampling the realizations at $\tau_0/40$ using linear interpolation between the realization points. If the sampling was done at τ_0/N_0, then the minimum fade duration would be just τ_0/N_0 (i.e., one sample interval of the realization). Thus a conclusion that can be drawn is that the second order statistics of deep fades in a realization improve as the number of samples between realization points increases. This is not to say that the durations of 50 or 60 dB or deeper fades can be accurately reproduced from $100\tau_0$ realizations generated with $\Delta t = \tau_0/10$. However, the duration of 30 dB fades or less can be reproduced from these realizations with appropriate sampling of the realizations.

Measured and ensemble mean fade separations for these three cases are shown in Figures 22a, 22b, and 22c. It is likely that $100\tau_0$ realizations will not have two fades below the 25 or 30 dB level so a fade separation measurement cannot be made. Thus the error bars at these levels are large and the measured separation for 30 dB fades is low by a factor of almost 10. For fades less than 20 dB or so, the $100\tau_0$ realizations reproduce the ensemble values for fade separation. If accurate separation statistics of deeper fades are of concern, however, then $100\tau_0$ realizations may not be adequate. It is clear from the plots in Figure 22 that $400\tau_0$ realizations are necessary to reproduce accurately the mean separation of 30 dB fades.

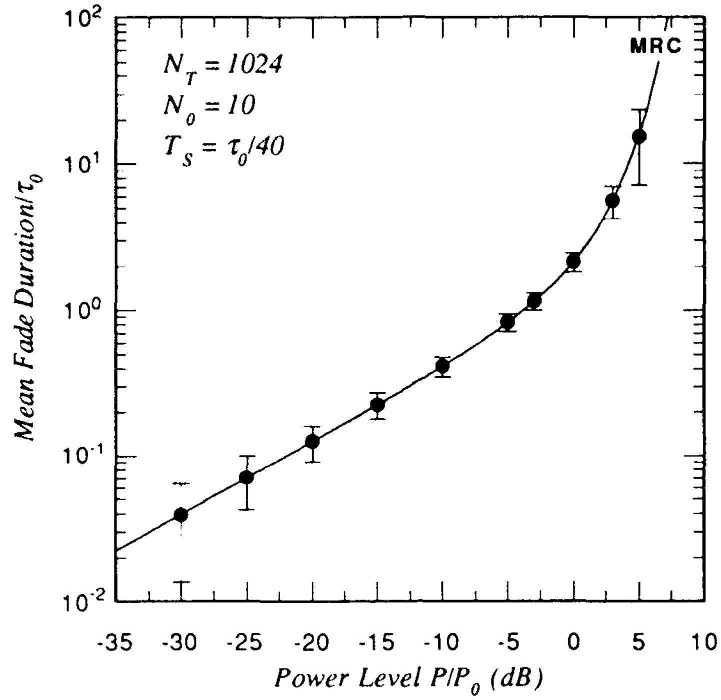

Figure 21a. Mean fade duration of $100\tau_0$ realizations.

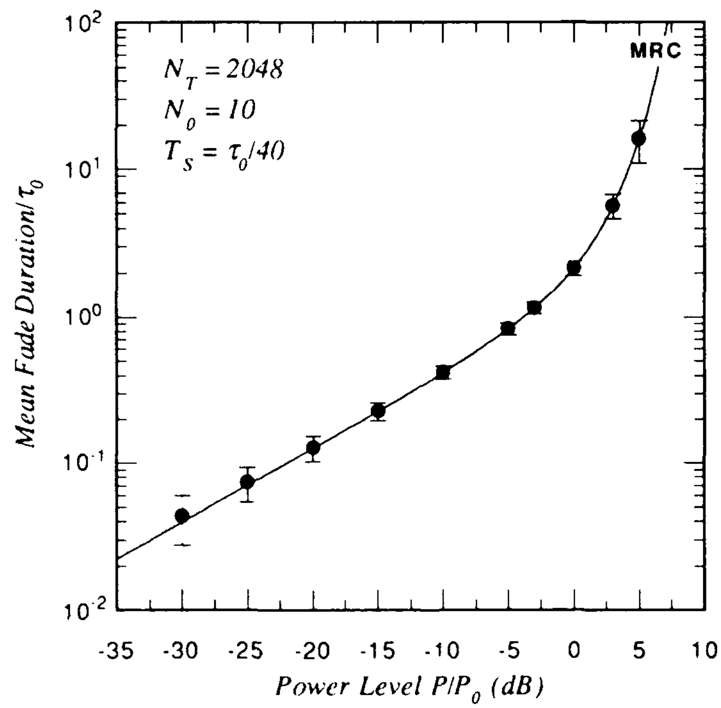

Figure 21b. Mean fade duration of $200\tau_0$ realizations.

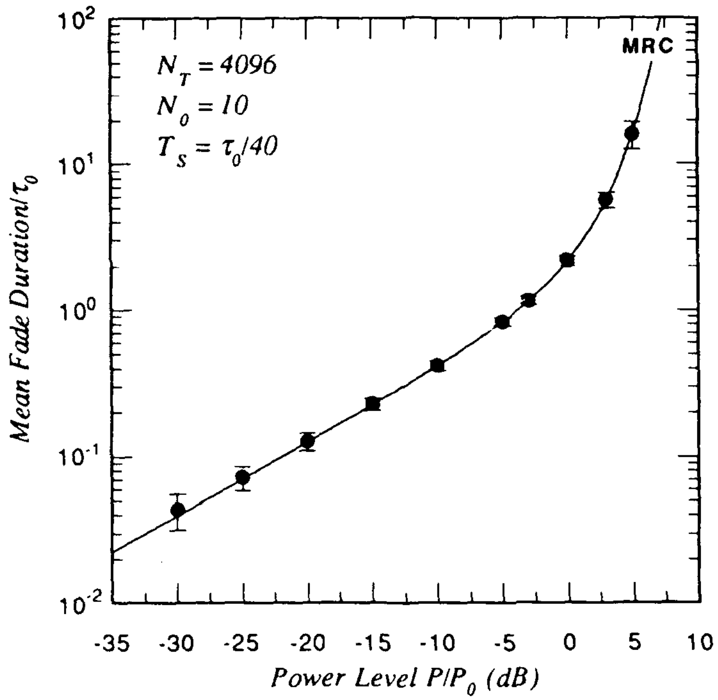

Figure 21c. Mean fade duration of $400\tau_0$ realizations.

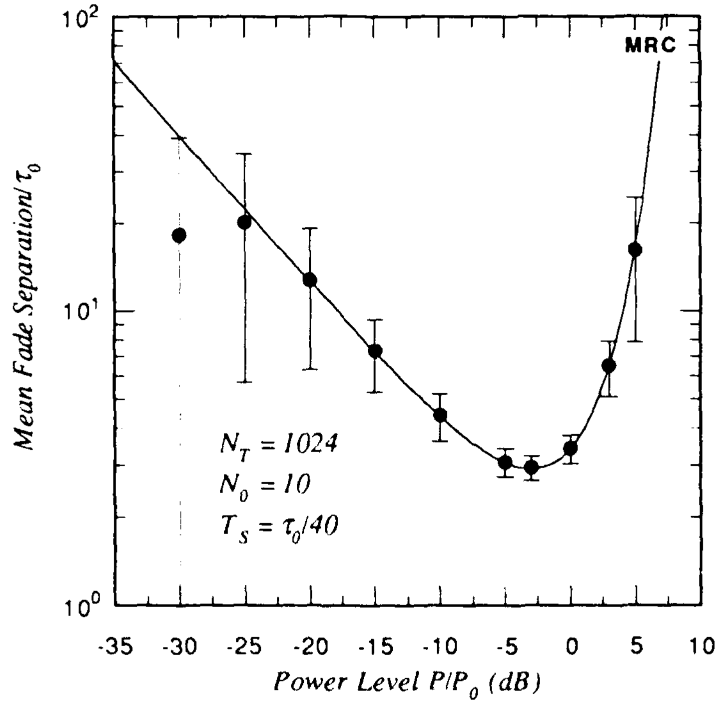

Figure 22a. Mean fade separation of $100\tau_0$ realizations.

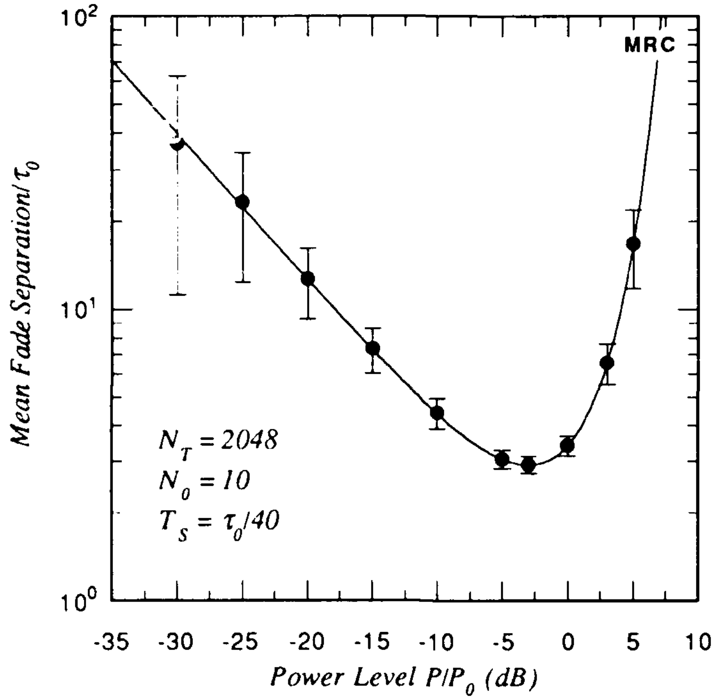

Figure 22b. Mean fade separation of $200\tau_0$ realizations.

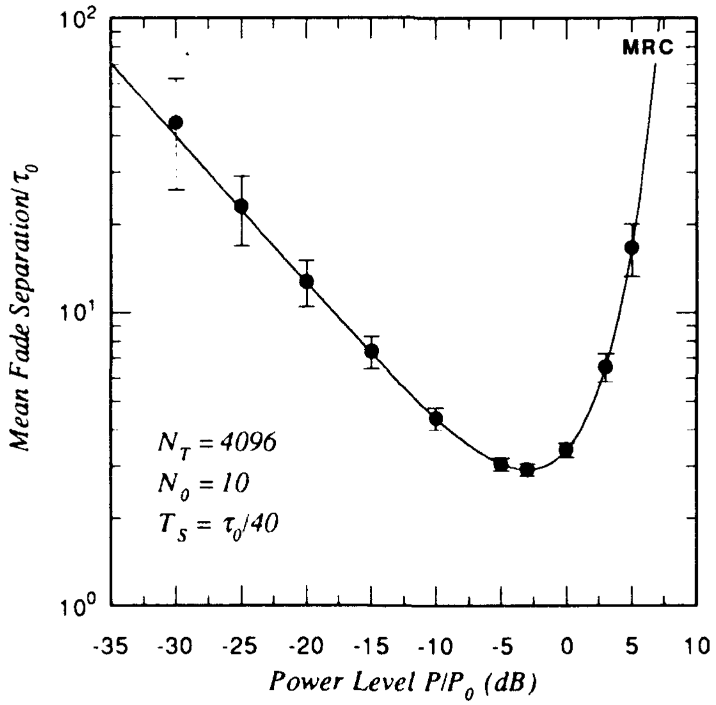

Figure 22c. Mean fade separation of $400\tau_0$ realizations.

APPENDIX D
LISTINGS OF ACIRF AND CIRF FORTRAN CODES

Listings of ACIRF and CIRF Fortran code are in this appendix in Section D.1. The "INCLUDE" files that are used to set array sizes are described in Section D.2. Limits on array sizes are also discussed in second section.

D.1 FORTRAN CODE.

The routines used by ACIRF and CIRF are listed in Tables 12 and 13 respectively. Common routines used by both codes are listed in Table 14. A short description of the processing performed by each routine is included in the tables.

Table 12. ACIRF Fortran code routines.

Routine	*Processing*
ACIRF	• Reads in channel, antenna, and realization parameters • Calculates ensemble filtered values of channel parameters • Computes grid sizes and checks for consistency • Prints input data and ensemble channel parameters • Initializes binary files for realizations of impulse response function • Calls CHANL1 and MEASR1
CHANL1	• Establishes angular grid • Calls FILAMP for power in Doppler frequency grid cells • Assigns angular grid cells to delay bins using $\boldsymbol{K}-\tau$ delta function and generates random voltage in each $\boldsymbol{K}-\tau$ cell • Performs DFT from angle to position of each antenna • Performs FFT from Doppler frequency to time for each delay bin • Writes impulse response function using WRITER
FILAMP	• Calculates K_x-K_y grid power using a fine K_p-K_q grid
MEASR1	• Reads impulse response functions using READER • Measures and prints statistics of impulse response functions – amplitude moments versus delay – decorrelation time and frequency selective bandwidth – cross correlation of impulse response function for all antennas
READER	• Reads impulse response function out of binary files (ACIRF version)
TAUGD1	• Computes energy in delay grid for each antenna using PIRF1 • Stops ACIRF if too little delay, advises grid change if too much delay
WRITER	• Writes impulse response function into binary files (ACIRF version)
ARCTAN	• Inverse tangent function – used to compute power in delay grid
BIOEXP	• Modified Bessel function $e^{-x}I_0(x)$ – used in PIRF1
PIRF1	• Computes power impulse response function

Table 13. CIRF Fortran code routines.

Routine	Processing
CIRF	• Reads in channel and realization parameters • Computes grid sizes and checks for consistency • Prints input data and ensemble channel parameters • Initializes binary file for realizations of impulse response function • Calls channel models: – CHANL0 – AWGN channel – CHANL2 – Frozen-in model – CHANL3 – Turbulent model – CHANL4 – Flat fading model • Calls MEASR2
CHANL0	• Writes constant impulse response function using WRITER
CHANL2	• Loops over delay bins • Computes power and random voltage in $\omega_D - \tau$ grid cells using TAUPSD • Performs FFT from Doppler frequency to time for each delay bin • Writes impulse response function using RESORT
RESORT	• Resorts impulse response function for frozen-in model from all times at each delay to all delays at each time and calls WRITER
TAUPSD	• Power density in in $\omega_D - \tau$ grid cells using WITWER
WITWER	• Wittwer's G function – DNA 5662D p. 34
CHANL3	• Initializes and warms-up f^{4} filters • Computes random time samples for each delay bin • Writes impulse response function using WRITER
CHANL4	• Initializes and warms-up f^{4} filter • Computes random time samples • Writes impulse response function using WRITER
TAUGD2	• Computes energy in delay grid using PIRF2 • Stops CIRF if too little energy • Recommends delay grid change if too much delay
MEASR2	• Measures and prints statistics of impulse response function – amplitude moments versus delay – decorrelation time and frequency selective bandwidth
READER	• Reads impulse response function out of binary files (CIRF version)
WRITER	• Writes impulse response function into binary files (CIRF version)
PIRF2	• Power impulse response function for frozen-in model

Table 14. ACIRF and CIRF shared Fortran routines.

Routine	*Processing*
FFT	• Fast Fourier transform
PREPRC	• Strips comment lines out of input files
LENGTH	• Computes the length of a character string – used in PREPRC
RHO	• Measures e^{-1} point of autocorrelation function – used in MEASRn
AWGN	• Generates zero-mean, unity-power complex white Gaussian noise
ERFC	• Complimentary error function
RANDOM	• Machine-independent uniformly distributed random number generator
SIMPSN	• Simpson's rule integration

Table 15 gives a cross-reference matrix (code name, table number, and page number) for the following tables that contain listings of the ACIRF and CIRF Fortran code.

Table 15. Fortran listing cross-reference.

ACIRF			*CIRF*		
Routine	*Table No.*	*Page No.*	*Routine*	*Table No.*	*Page No.*
ACIRF	16	108	AWGN	43	170
ARCTAN	23	143	CHANL0	27	150
AWGN	43	170	CHANL2	28	151
BIOEXP	24	143	CHANL3	32	155
CHANL1	17	120	CHANL4	33	157
ERFC	44	171	CIRF	26	145
FFT	39	168	ERFC	44	171
FILAMP	18	128	FFT	39	168
LENGTH	41	169	LENGTH	41	169
MEASR1	19	132	MEASR2	35	160
PIRF1	25	144	PIRF2	38	167
PREPRC	40	169	PREPRC	40	169
RANDOM	45	171	RANDOM	45	171
READER	20	137	READER	36	164
RHO	42	170	RESORT	29	153
SIMPSN	46	172	RHO	42	170
TAUGD1	21	139	SIMPSN	46	172
WRITER	22	141	TAUGD2	34	158
			TAUPSD	30	154
			WITWER	31	154
			WRITER	37	166

Table 16. ACIRF Fortran code.

```
      PROGRAM ACIRF
      PARAMETER (VERSON=3.51)
C
C******************** VERSION 3.51 *** 24 APR 1991 **************************
C
C     THIS PROGRAM GENERATES REALIZATIONS OF THE CHANNEL IMPULSE RESPONSE
C     FUNCTION AT THE OUTPUTS OF MULTIPLE ANTENNAS.  TEMPORAL VARIATIONS
C     OF THE IMPULSE RESPONSE FUNCTIONS ARE OBTAINED USING THE GENERAL MODEL.
C
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C
C*********************  SET ARRAY SIZES  ************************************
C
C  MTENNA = MAXIMUM NUMBER OF ANTENNAS
C  MDELAY = MAXIMUM NUMBER OF DELAY SAMPLES
C  MKX    = MAXIMUM NUMBER OF KX SAMPLES
C  MKY    = MAXIMUM NUMBER OF KY SAMPLES
C  MTIMES = MAXIMUM NUMBER OF TIME SAMPLES
C  MAXBUF = MAXIMUM NUMBER OF WORDS IN A BUFFER
C           MAXIMUM RECORD SIZE = MAXBUF + 1
C
      INCLUDE 'ACIRF.SIZ'

C
C**************************  INPUT DATA  ************************************
C
C  CHANNEL PARAMETERS
C
C    F0      = FREQUENCY SELECTIVE BANDWIDTH (HZ)
C    TAU0    = DECORRELATION TIME (SECONDS)
C    DLX     = X-DIRECTION DECORRELATION DISTANCE (METERS)
C    DLY     = Y-DIRECTION DECORRELATION DISTANCE (METERS)
C    CXT     = TIME-X CROSS CORRELATION COEFFICIENT
C    CYT     = TIME-Y CROSS CORRELATION COEFFICIENT
C
C  ANTENNA PARAMETERS
C
C    NUMANT  = NUMBER OF ANTENNAS (MUST BE >=1 AND <= MTENNA)
C    FREQC   = CARRIER FREQUENCY (HZ)
C
C  FOR EACH ANTENNA:
C
```

Table 16. ACIRF Fortran code (Continued).

```
C    BWV     = 3 DB ANTENNA BEAMWIDTH ABOUT V-AXIS (DEGREES)
C    BWU     = 3 DB ANTENNA BEAMWIDTH ABOUT U-AXIS (DEGREES)
C    UPOS    = U POSITION OF ANTENNA CENTER (M)
C    VPOS    = V POSITION OF ANTENNA CENTER (M)
C    ROT     = ROTATION ANGLE BETWEEN X-AXIS AND U-AXIS (DEGREES)
C    ELV     = ELEVATION POINTING ANGLE MEASURED FROM LINE-OF-SIGHT (DEGREES)
C    AZM     = AZIMUTH   POINTING ANGLE MEASURED FROM U-AXIS (DEGREES)
C
C    NOTES:   BEAMWIDTHS ARE FULL WIDTH AT HALF MAXIMUM
C             ANGLES DEFINED BY THE FOLLOWING:
C                 X cross U   = LOS SIN(RAD(ROT))
C                 X dot U     = COS(RAD(ROT))
C                 LOS cross X = Y
C                 LOS cross U = V
C                 PNT dot LOS = COS(RAD(ELV))
C                 P           = PNT - LOS ( PNT dot LOS )
C                 U cross P   - LOS SIN(RAD(AZM))
C                 P dot U     = COS(RAD(AZM))
C             WHERE
C                 X,Y, LOS ARE UNIT VECTORS IN A RIGHT HANDED CARTESIAN
C                   COORDINATE SYSTEM IN WHICH THE PROPAGATION PARAMETERS
C                   ARE DEFINED,
C                 U,V, LOS ARE UNIT VECTORS IN A RIGHT HANDED CARTESIAN
C                   COORDINATE SYSTEM IN WHICH THE ANTENNA PARAMETERS ARE
C                   DEFINED,
C                 LOS IS A UNIT VECTOR PARALLEL TO THE PROPAGATION LINE OF
C                   SIGHT
C                 PNT IS A UNIT VECTOR POINTING IN THE ANTENNA BORE SIGHT
C                   DIRECTION
C
C  REALIZATION PARAMETERS
C
C    KASE    = KASE NUMBER OF REALIZATION
C    IDENT   = ALPHANUMERIC IDENTIFIER FOR REALIZATION
C    DELTAU  = DELAY SAMPLE SIZE (SEC)
C    NDELAY  = NUMBER OF DELAY SAMPLES
C    NKX     = NUMBER OF KX SAMPLES
C    NKY     = NUMBER OF KY SAMPLES
C    NTIMES  = NUMBER OF TIME SAMPLES
C    IJSEED  = INITIAL RANDOM NUMBER SEED
C    NO      = NUMBER OF SAMPLES PER DECORRELATION TIME
C
C    NOTES:  CASE NUMBER FOR IDENTIFICATION ONLY
C            IDENT MUST BE <= 80 CHARACTERS
C            NDELAY MUST BE <= MDELAY
C            NKX DEFAULT = 32 (MUST BE >= 32 AND <= MKX)
C            NKY DEFAULT = 32 (MUST BE >= 32 AND <= MKY)
C            NTIMES DEFAULT = 1024
C            IJSEED DEFAULT = 9771975
C            NO DEFAULT = 10 (MUST BE >= 10)
C
C******************************************************************************
C
      CHARACTER NAME*9,IDENT*80
      LOGICAL STOPER
      COMMON/ANTENA/NUMANT,XPOS(MTENNA),YPOS(MTENNA)
```

Table 16. ACIRF Fortran code (Continued).

```
      COMMON/ANTNUM/IANT
      COMMON/CHANEL/DLX,DLY,F0,CXT,CYT
      COMMON/ENSMBL/CROSS(MTENNA,MTENNA),PA(MTENNA),SLOSDB(MTENNA),
     1  FA(MTENNA),TAUA(MTENNA)
      COMMON/PROFIL/BEAM1(MTENNA),BEAM2(MTENNA),BEAM3(MTENNA),
     1  PNTAKX(MTENNA),PNTAKY(MTENNA)
      COMMON/GRDSIZ/DELTAU,DELTKX,DELTKY,TAU0DF
      COMMON/HEADER/RDATA1(NDATA1),RDATA2(MTENNA,NDATA2)
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/INTGRL/NTASUB
      COMMON/POWERS/G(MDELAY,MTENNA),GRDPOW(3,MTENNA)
      COMMON/POWIRF/CPIRF(5,MTENNA),G0(MTENNA)
      COMMON/RANSED/ISEED,JSEED
      COMMON/REALZN/NKX,NKY,NDELAY,NTIMES,N0,NFMAX
      COMPLEX CROSS
      DIMENSION BWU(MTENNA),BWV(MTENNA),UPOS(MTENNA),VPOS(MTENNA),
     1  ROT(MTENNA),ANGLSS(MTENNA),ANTVCX(MTENNA),ANTVCY(MTENNA),
     2  DETQ(MTENNA),AZM(MTENNA),ELV(MTENNA),
     3  QXX(MTENNA),QYY(MTENNA),QXY(MTENNA),TRCQ(MTENNA)
      PARAMETER (PI=3.141592654,TWOPI=2.0*PI)
      PARAMETER (SMALL=1.0E-10)
      DATA STOPER/.TRUE./
      RAD(X) = PI*X/180.0
C
C******  SET CONSTANTS AND FIXED PARAMETERS
C
      NTASUB = 100
      INPUT  = 1
      IREAD  = 2
      IWRITE = 3
      IOFILE = 11
C
C******************************************************************************
C
      OPEN(UNIT=IREAD ,FILE='ACIRF.DAT',STATUS='OLD')
      OPEN(UNIT=IWRITE,FILE='ACIRF.OUT',STATUS='NEW')
      CALL PREPRC(IREAD,INPUT)
      CLOSE(UNIT=IREAD)
C
C******  READ INPUT DATA AND SET DEFAULT VALUES
C
C  CASE NUMBER AND IDENTIFICATION
C
      READ(INPUT,*)KASE,STOPER
      IDENT = ' '
      READ(INPUT,*)IDENT
      MDENT = LENGTH(IDENT)
C
C  CHANNEL PARAMETERS
C
      READ(INPUT,*)F0,TAU0,DLX,DLY,CXT,CYT
      IF(CXT**2+CYT**2 .GT. 0.9998)THEN
          WRITE(*,4000)
 4000     FORMAT(2X,'****** INPUT ERROR ******')
          WRITE(*,4020)CXT,CYT
```

Table 16. ACIRF Fortran code (Continued).

```
 4020     FORMAT(2X,'CXT**2 + CYT**2 MUST BE <= 0.9998',/2X,1P,
     1      'CXT = ',E12.4,'  CYT = ',E12.4)
          STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  ANTENNA PARAMETERS
C
      READ(INPUT,*)NUMANT,FREQC
      CLAMDA = 2.997925E8/FREQC
C
C  THE NUMBER OF ANTENNAS MUST BE >= 1 AND <= MTENNA
C
      IF(NUMANT .LT. 1 .OR. NUMANT .GT. MTENNA)THEN
          WRITE(*,4000)
          WRITE(*,4030)NUMANT,MTENNA
 4030     FORMAT(2X,'NUMANT MUST BE >= 1 AND <= MTENNA',/2X,
     1      'NUMANT = ',I10/2X,'MTENNA = ',I10)
          STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  READ ANTENNA BEAMWIDTHS, POSITIONS, AND POINTING ANGLES
C
      DO 5 IANT=1,NUMANT
          READ(INPUT,*)BWU(IANT),BWV(IANT),UPOS(IANT),VPOS(IANT),
     1      ROT(IANT),ELV(IANT),AZM(IANT)
    5 CONTINUE
C
C  REALIZATION PARAMETERS
C
      NKX = 32
      NKY = 32
      NO  = 10
      NTIMES = 1024
      IJSEED = 9771975
      READ(INPUT,*)NDELAY,DELTAU,NTIMES,NKX,NKY,IJSEED,NO
      CLOSE(UNIT=INPUT,STATUS='DELETE')
      JSEED = IAND(IJSEED,65535)
      ISEED = IAND(ISHFT(IJSEED,-16),65535)
C
C  THERE MUST BE AT LEAST 10 SAMPLES PER DECORRELATION DISTANCE OR TIME
C
      IF(NO .LT. 10)THEN
          WRITE(*,4000)
          WRITE(*,4040)NO
 4040     FORMAT(2X,'NO MUST BE >= 10',/2X,'NO = ',I10)
          IF(STOPER)STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  THERE MUST BE AT LEAST 100 DECORRELATION TIMES IN EACH REALIZATION
C
      NTMIN = 100*NO
      IF(NTIMES .LT. NTMIN)THEN
          WRITE(*,4000)
          WRITE(*,4050)NTIMES,NTMIN,MTIMES
```

Table 16. ACIRF Fortran code (Continued).

```
 4050     FORMAT(2X,'THERE MUST BE AT LEAST 100 DECORRELATION TIMES ',
     1       'IN EACH REALIZATION',/2X,'NTIMES = ',I10/2X,
     2       'REQUIRED NTIMES = ',I10/2X,'MAXIMUM NTIMES = ',I10)
          STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  NDELAY MUST BE >= 1 AND <= MDELAY
C
      IF(NDELAY .LT. 1 .OR. NDELAY .GT. MDELAY)THEN
          WRITE(*,4000)
          WRITE(*,4060)NDELAY,MDELAY
 4060     FORMAT(2X,'NDELAY MUST BE >= 1 AND <= MDELAY',/2X,
     1       'NDELAY = ',I10/2X,'MDELAY = ',I10)
          STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  NTIMES MUST BE >=1 AND <= MTIMES
C
      IF(NTIMES .LT. 1 .OR. NTIMES .GT. MTIMES)THEN
          WRITE(*,4000)
          WRITE(*,4070)NTIMES,MTIMES
 4070     FORMAT(2X,'NTIMES MUST BE >= 1 AND <= MTIMES',/2X,
     1       'NTIMES = ',I10/2X,'MTIMES = ',I10)
          STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  NKX MUST BE >= 32 AND <= MKX
C
      IF(NKX .LT. 32 .OR. NKX .GT. MKX)THEN
          WRITE(*,4000)
          WRITE(*,4080)NKX,MKX
 4080     FORMAT(2X,'NKX MUST BE >= 32 AND <= MKX',/2X,
     1       'NKX = ',I10/2X,'MKX = ',I10)
          STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  NKY MUST BE >= 32 AND <= MKY
C
      IF(NKY .LT. 32 .OR. NKY .GT. MKY)THEN
          WRITE(*,4000)
          WRITE(*,4090)NKY,MKY
 4090     FORMAT(2X,'NKY MUST BE >= 32 AND <= MKY',/2X,
     1       'NKY = ',I10/2X,'MKY = ',I10)
          STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C******  CALCULATE ENSEMBLE CHANNEL PARAMETERS OUT OF ANTENNAS
C
      WAVNUM = TWOPI/CLAMDA
      DETDL = (DLX*DLY/4.0)**2
      TRCDL2 = (DLX**4 + DLY**4)/16.0
      TOMIN = 1.0E30
      TOMAX = 0.0
      FMAX  = 0.0
```

Table 16. ACIRF Fortran code (Continued).

```
      DO 10 IANT=1,NUMANT
          RSIN = SIN(RAD(ROT(IANT)))
          RCOS = COS(RAD(ROT(IANT)))
C
C   ANTENNA BEAM PROFILE PARAMETERS
C
          THETOU = RAD(BWU(IANT))
          THETOV = RAD(BWV(IANT))
          ALFAU =  ALOG(2.0)*(CLAMDA/(PI*THETOU))**2
          ALFAV =  ALOG(2.0)*(CLAMDA/(PI*THETOV))**2
          BEAM1(IANT) = ALFAU*RCOS**2 + ALFAV*RSIN**2
          BEAM2(IANT) = ALFAU*RSIN**2 + ALFAV*RCOS**2
          BEAM3(IANT) = 2.0*(ALFAU - ALFAV)*RSIN*RCOS
C
C   SCATTERING LOSS
C
          QXX(IANT) = DLX**2/4.0 + BEAM1(IANT)
          QYY(IANT) = DLY**2/4.0 + BEAM2(IANT)
          QXY(IANT) = BEAM3(IANT)/2.0
          DETQ(IANT) = QXX(IANT)*QYY(IANT) - QXY(IANT)**2
          TRCQ(IANT) = QXX(IANT) + QYY(IANT)
          PANGVU = WAVNUM*SIN(RAD(ELV(IANT)))*COS(RAD(AZM(IANT)))
          PANGVV = WAVNUM*SIN(RAD(ELV(IANT)))*SIN(RAD(AZM(IANT)))
          PNTAKX(IANT) = PANGVU*RCOS - PANGVV*RSIN
          PNTAKY(IANT) = PANGVU*RSIN + PANGVV*RCOS
          ANTVCX(IANT) = RCOS*ALFAU*PANGVU - RSIN*ALFAV*PANGVV
          ANTVCY(IANT) = RSIN*ALFAU*PANGVU + RCOS*ALFAV*PANGVV
          ANGLSS(IANT) = (QYY(IANT)*ANTVCX(IANT)**2 -
     1      2.0*QXY(IANT)*ANTVCX(IANT)*ANTVCY(IANT) +
     2      QXX(IANT)*ANTVCY(IANT)**2)/DETQ(IANT)
          ARG = BEAM1(IANT)*PNTAKX(IANT)**2 +
     1          BEAM2(IANT)*PNTAKY(IANT)**2 +
     2          BEAM3(IANT)*PNTAKX(IANT)*PNTAKY(IANT)
          G0(IANT) = EXP(-ARG)
          PA(IANT) = SQRT(DETDL/DETQ(IANT))*EXP(ANGLSS(IANT))*
     1      G0(IANT)
          IF(PA(IANT) .LE. SMALL)THEN
            WRITE(*,4125)IANT,PA(IANT)
 4125       FORMAT(2X,'**********  WARNING  **********'/2X,
     1        'SCATTERING LOSS OF ANTENNA ',I2,' EXCEEDS 100 DB'
     2        'RECEIVED POWER = ',1PE10.3/2X,
     3        'ACIRF MAY NOT BE ABLE TO HANDLE THIS CASE')
          END IF
          SLOSDB(IANT) = -10.0*ALOG10(PA(IANT))
          GRDPOW(3,IANT) = PA(IANT)
C
C   FREQUENCY SELECTIVE BANDWIDTH
C
          ANTVEC = SQRT(ANTVCX(IANT)**2 + ANTVCY(IANT)**2)
          FA(IANT) = F0*SQRT(TRCDL2)/DETDL/SQRT(4.0*ANGLSS(IANT)*
     1      ((TRCQ(IANT)/DETQ(IANT))**2 - 1.0/DETQ(IANT)) +
     2      (TRCQ(IANT)**2 - 4.0*TRCQ(IANT)*ANTVEC**2)
     3      /DETQ(IANT)**2 - 2.0/DETQ(IANT))
C
C   DECORRELATION TIME
C
```

Table 16. ACIRF Fortran code (Continued).

```
        DX = CXT*DLX
        DY = CYT*DLY
        TAUA(IANT) = TAU0*SQRT(DETQ(IANT)/(DETQ(IANT)*
     1    (1.0-CXT**2-CYT**2)+(QYY(IANT)*DX**2+QXX(IANT)*
     2    DY**2)/4.0-DX*DY*QXY(IANT)/2.0))
        IF(TAUA(IANT) .LT. TOMIN)TOMIN = TAUA(IANT)
        IF(TAUA(IANT) .GT. TOMAX)TOMAX = TAUA(IANT)
C
C  MEAN DOPPLER FREQUENCY AND MAXIMUM REQUIRED DOPPLER FREQUENCY
C
        DOPLER =((ANTVCY(IANT)*QXX(IANT)*DY +
     1    ANTVCX(IANT)*QYY(IANT)*DX -
     2    (ANTVCX(IANT)*DY+ANTVCY(IANT)*DX)*QXY(IANT))/
     3    DETQ(IANT))/(TWOPI*TAU0)
        DOPMAX = ABS(DOPLER) + 2.4612/(PI*TAUA(IANT))
        FMAX=AMAX1(FMAX,DOPMAX)
C
C  POWER IMPULSE RESPONSE FUNCTION PARAMETERS
C
        CPIRF(1,IANT) = TRCQ(IANT)*SQRT(TRCDL2/2.0)/DETDL/2.0
        CPIRF(2,IANT) = SQRT(TRCDL2*ABS(TRCQ(IANT)**2 -
     1    4.0*DETQ(IANT))/2.0)/DETDL/2.0
        CPIRF(3,IANT) = 2.0*ANTVEC*(TRCDL2/2.0)**(0.25)/
     1    SQRT(DETDL)
        IF(ANTVEC .NE. 0.0) THEN
            CPIRF(4,IANT) = 2.0*ARCTAN(ANTVCY(IANT),ANTVCX(IANT))
     1      - ARCTAN(2.0*QXY(IANT),(QXX(IANT)-QYY(IANT))) + PI
        ELSE
            CPIRF(4,IANT) = -ARCTAN(2.0*QXY(IANT),QXX(IANT) -
     1      QYY(IANT)) + PI
        END IF
        CPIRF(5,IANT) = SQRT(TRCDL2/2.0/DETDL)
   10 CONTINUE
C
C  ANTENNA CENTER POSITIONS IN X-Y COORDINATE SYSTEM
C
      XMAX = -1.0E30
      YMAX = -1.0E30
      XMIN = 0.0
      YMIN = 0.0
      DO 20 IANT=1,NUMANT
          RSIN = SIN(RAD(ROT(IANT)))
          RCOS = COS(RAD(ROT(IANT)))
          XPOS(IANT) = UPOS(IANT)*RCOS - VPOS(IANT)*RSIN
          YPOS(IANT) = UPOS(IANT)*RSIN + VPOS(IANT)*RCOS
          XMAX = AMAX1(XMAX,XPOS(IANT))
          XMIN = AMIN1(XMIN,XPOS(IANT))
          YMAX = AMAX1(YMAX,YPOS(IANT))
          YMIN = AMIN1(YMIN,YPOS(IANT))
   20 CONTINUE
C
C  ANTENNA OUTPUT VOLTAGE CROSS CORRELATION
C
      IF(NUMANT .GT. 1)THEN
          DO 40 IANT=1,NUMANT
```

Table 16. ACIRF Fortran code (Continued).

```
              DO 30 JANT=IANT,NUMANT
                  DX = XPOS(IANT) - XPOS(JANT)
                  DY = YPOS(IANT) - YPOS(JANT)
                  QXXMN = (QXX(IANT) + QXX(JANT))/2.0
                  QXYMN = (QXY(IANT) + QXY(JANT))/2.0
                  QYYMN = (QYY(IANT) + QYY(JANT))/2.0
                  DETQMN = QXXMN*QYYMN - QXYMN**2
                  ANVXMN = (ANTVCX(IANT) + ANTVCX(JANT))/2.0
                  ANVYMN = (ANTVCY(IANT) + ANTVCY(JANT))/2.0
                  ANTARG = (QYYMN*ANVXMN**2 - 2.0*QXYMN*ANVXMN
     1                *ANVYMN + QXXMN*ANVYMN**2)/DETQMN
                  ARG = (QYYMN*DX**2 - 2.0*QXYMN*DX*DY
     1                + QXXMN*DY**2)/DETQMN/4.0
                  PHASE = (ANVXMN*(QXYMN*DY - QYYMN*DX)
     1                + ANVYMN*(QXYMN*DX - QXXMN*DY))/DETQMN
                  CROSS(JANT,IANT) =
     1                SQRT(SQRT(DETQ(IANT)*DETQ(JANT)))/SQRT(DETQMN)*
     2                EXP(ANTARG-(ANGLSS(IANT)+ANGLSS(JANT))/2.0)*
     3                CEXP(CMPLX(-ARG,PHASE))
                  CROSS(IANT,JANT) = CROSS(JANT,IANT)
   30         CONTINUE
   40     CONTINUE
      END IF
C
C  FIND LIMITS ON ANGULAR GRID TO ENCOMPASS 99.9 PERCENT OF ENERGY
C
      PTXMAX = 0.0
      PTYMAX = 0.0
      DO 50 IANT=1,NUMANT
          AKXMAX = ABS((ANTVCX(IANT)*QYY(IANT) -
     1      ANTVCY(IANT)*QXY(IANT))/
     2      DETQ(IANT)) + 2.5895*SQRT(QYY(IANT)/DETQ(IANT))
          IF(AKXMAX .GT. PTXMAX)PTXMAX = AKXMAX
          AKYMAX = ABS((ANTVCY(IANT)*QXX(IANT) -
     1      ANTVCX(IANT)*QXY(IANT))/
     2      DETQ(IANT)) + 2.5895*SQRT(QXX(IANT)/DETQ(IANT))
          IF(AKYMAX .GT. PTYMAX)PTYMAX = AKYMAX
   50 CONTINUE
C
C******  CALCULATE GRID SIZES AND CHECK FOR CONSISTENCY
C
C  DELTA TIME GRID SIZE
C
      DELTAT = TOMIN/NO
      IF(NTIMES*DELTAT .LT. 100.0*TOMAX)THEN
          WRITE(*,4130)
 4130     FORMAT(2X,'****** GRID SIZE ERROR ******')
          NTMIN = INT(100.0*TOMAX/DELTAT) + 1
          WRITE(*,4140)NTIMES,NTMIN,MTIMES
 4140     FORMAT(2X,'THERE ARE NOT 100 DECORRELATION TIMES IN ',
     1      'REALIZATION OF ANTENNA WITH LARGEST TAUA'/2X,
     2      'INCREASE NTIMES'/2X,'NTIMES = ',I10/2X,
     3      'REQUIRED NTIMES = ',I10/2X,'MAXIMUM NTIMES = ',I10)
          IF(STOPER)STOP ' ACIRF EXECUTION TERMINATED'
      END IF
      TAUODF = TAU0/(NTIMES*DELTAT)
```

Table 16. ACIRF Fortran code (Continued).

```
      NFMAX = INT(FMAX*NTIMES*TOMIN/NO) + 1
      NFREQ = 2*NFMAX
      IF(NFMAX .GT. NTIMES/2)THEN
          JJ = INT(FLOAT(2*NFMAX*NO)/FLOAT(NTIMES))+1
          WRITE(*,4150)JJ
 4150     FORMAT(2X,'***** TOO FEW TIMES PER TAU0 *****',/,
     1      4X,'NO MUST BE >= ',I5)
         STOP ' ACIRF EXECUTION TERMINATED'
      END IF
C
C  DELTA KX`GRID SIZE
C
      EXTREM = ABS(XMAX-XMIN)
      ALIAS  = PI/AMAX1(EXTREM,SMALL)
      DELTKX = 2.0*PTXMAX/(NKX-1)
C
C  DELTAKX TOO LARGE - POSSIBLE ALIASING OF OUTPUTS OF ANTENNAS
C
      IF(DELTKX .GT. ALIAS)THEN
          NKXMIN = 2.0*PTXMAX/ALIAS
          WRITE(*,4130)
          WRITE(*,4170)NKX,NKXMIN,MKX
 4170     FORMAT(2X,'NKX TOO SMALL - ALIASING OF ANTENNA OUTPUTS'
     1      /2X,'NKX = ',I10/2X,'REQUIRED NKX = ',I10/2X,
     2      'MAXIMUM ALLOWED NKX = '.I10)
          IF(STOPER)STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  DELTA KY GRID SIZE
C
      EXTREM = ABS(YMAX-YMIN)
      ALIAS  = PI/AMAX1(EXTREM,SMALL)
      DELTKY = 2.0*PTYMAX/(NKY-1)
C
C  DELTAKY TOO LARGE - POSSIBLE ALIASING OF OUTPUTS OF ANTENNAS
C
      IF(DELTKY .GT. ALIAS)THEN
          NKYMIN = 2.0*PTYMAX/ALIAS
          WRITE(*,4130)
          WRITE(*,4180)NKY,NKYMIN,MKY
 4180     FORMAT(2X,'NKY TOO SMALL - ALIASING OF ANTENNA OUTPUTS'
     1      /2X,'NKY = ',I10/2X,'REQUIRED NKY = ',I10/2X,
     2     'MAXIMUM ALLOWED NKY = ',I10)
          IF(STOPER)STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C  CHECK DELAY GRID SIZE
C
      CALL TAUGD1(NDRQD)
C
C******  PRINT INPUT DATA AND ENSEMBLE CHANNEL PARAMETERS
C
      WRITE(IWRITE,2000)VERSON,KASE
 2000 FORMAT(2X,'ACIRF CHANNEL SIMULATION VERSION ',F6.2/4X,
     1  'CASE NUMBER ',I10)
```

Table 16. ACIRF Fortran code (Continued).

```
      WRITE(IWRITE,2001)
2001  FORMAT(4X,'TEMPORAL VARIATION FROM GENERAL MODEL')
      WRITE(IWRITE,2009) IDENT(1:MDENT)
2009  FORMAT(4X,'REALIZATION IDENTIFICATION:',/4X,A)
      WRITE(IWRITE,2010)F0,TAU0,DLX,DLY,CXT,CYT
2010  FORMAT(/2X,'CHANNEL PARAMETERS',/4X,
     1  'FREQUENCY SELECTIVE BANDWIDTH  (HZ) = ',1PE10.3/4X,
     2  'DECORRELATION TIME (SEC)            = ',E10.3/4X,
     3  'X DECORRELATION DISTANCE (M)        = ',E10.3/4X,
     4  'Y DECORRELATION DISTANCE (M)        = ',E10.3/4X,
     5  'TIME-X CORRELATION COEFFICIENT      = ',E10.3/4X,
     6  'TIME-Y CORRELATION COEFFICIENT      = ',E10.3)
      WRITE(IWRITE,2020)NDELAY,DELTAU,NTIMES,NFREQ,N0,NKX,NKY,IJSEED
2020  FORMAT(/2X,'REALIZATION PARAMETERS',/4X,
     1  'NUMBER OF DELAY SAMPLES             = ',I10/4X,
     2  'DELAY SAMPLE SIZE (SEC)             = ',1PE10.3/4X,
     3  'NUMBER OF TEMPORAL SAMPLES          = ',I10/4X,
     4  'NUMBER OF DOPPLER FREQUENCY SAMPLES = ',I10/4X,
     5  'NUMBER OF TEMPORAL SAMPLES PER TAU0 = ',I10/4X,
     6  'NUMBER OF KX SAMPLES                = ',I10/4X,
     7  'NUMBER OF KY SAMPLES                = ',I10/4X,
     8  'INITIAL RANDOM NUMBER SEED        = ',I12)
      IF(NDRQD .LT. NDELAY)WRITE(IWRITE,2025)NDRQD
2025  FORMAT(/2X,'**********   WARNING   **********'/2X,
     1  'ONLY ',I4,' DELAY BINS ARE REQUIRED'/2X,
     2  '************        ****************')
      WRITE(IWRITE,2040)NUMANT,FREQC
2040  FORMAT(/2X,'ANTENNA PARAMETERS',/4X,
     1  'NUMBER OF ANTENNAS       = ',I10/4X,
     2  'CARRIER FREQUENCY (HZ)   = ',1PE10.3)
      WRITE(IWRITE,2045)
2045  FORMAT(/2X,'ANTENNA BEAMWIDTHS, ROTATION ANGLES',
     1  ' AND POINTING ANGLES',//4X,
     2  ' N',3X,'BWU(DEG)',3X,'BWV(DEG)',3X,'ROT(DEG)',
     3  3X,'ELV(DEG)',3X,'AZM(DEG)')
      DO 70 IANT=1,NUMANT
          WRITE(IWRITE,2050)IANT,BWU(IANT),BWV(IANT),ROT(IANT),
     1      ELV(IANT),AZM(IANT)
2050      FORMAT(3X,I2,5(1X,F10.3))
   70 CONTINUE
      WRITE(IWRITE,2055)
2055  FORMAT(/2X,'ANTENNA POSITIONS IN U-V AND X-Y COORDINATES',
     1  //4X,'N',4X,'UPOS(M)',4X,'VPOS(M)',4X,
     2  'XPOS(M)',4X,'YPOS(M)')
      DO 75 IANT=1,NUMANT
          WRITE(IWRITE,2057)IANT,UPOS(IANT),VPOS(IANT),
     1      XPOS(IANT),YPOS(IANT)
2057      FORMAT(3X,I2,4(1X,1PE10.3))
   75 CONTINUE
      WRITE(IWRITE,2060)
2060  FORMAT(/2X,'ENSEMBLE CHANNEL PARAMETERS AT ANTENNA OUTPUTS'
     1  //4X,'N',3X,'LOSS(DB)',6X,'POWER',5X,'FA(HZ)',2X,
     2  'TAUA(SEC)')
      DO 80 IANT=1,NUMANT
```

Table 16. ACIRF Fortran code (Continued).

```
          WRITE(IWRITE,2065)IANT,SLOSDB(IANT),GRDPOW(3,IANT),
     1      FA(IANT),TAUA(IANT)
 2065     FORMAT(3X,I2,1X,F10.3,1X,F10.6,2(1X,1PE10.3))
   80 CONTINUE
C
C******  OPEN BINARY FILES FOR REALIZATIONS
C
      DO 100 II=1,NDATA1
          RDATA1(II) = 0.0
  100 CONTINUE
      DO 110 II=1,MTENNA
          DO 105 IJ=1,NDATA2
              RDATA2(II,IJ) = 0.0
  105     CONTINUE
  110 CONTINUE
      RDATA1( 1) = 2.0
      RDATA1( 2) = KASE
      RDATA1( 3) = FREQC
      RDATA1( 4) = TAU0
      RDATA1( 5) = F0
      RDATA1( 6) = DLX
      RDATA1( 7) = DLY
      RDATA1( 9) = 1.0
      RDATA1(13) = NTIMES*DELTAT
      RDATA1(14) = NTIMES
      RDATA1(15) = DELTAT
      RDATA1(16) = N0
      RDATA1(20) = NDELAY
      RDATA1(21) = DELTAU/2.0
      RDATA1(22) = DELTAU
      RDATA1(23) = ISEED
      RDATA1(24) = JSEED
      RDATA1(25) = MAXBUF
      RDATA1(26) = CXT
      RDATA1(27) = CYT
      RDATA1(28) = VERSON
      DO 120 IANT=1,NUMANT
          RDATA2(IANT, 1) = KASE
          RDATA2(IANT, 2) = 1.0
          RDATA2(IANT, 3) = TAUA(IANT)
          RDATA2(IANT, 4) = FA(IANT)
          RDATA2(IANT, 5) = DLX
          RDATA2(IANT, 6) = DLY
          RDATA2(IANT, 7) = NDELAY
          RDATA2(IANT, 8) = DELTAU
          RDATA2(IANT, 9) = NTIMES
          RDATA2(IANT,10) = NKX
          RDATA2(IANT,11) = NKY
          RDATA2(IANT,12) = IJSEED
          RDATA2(IANT,13) = ISEED
          RDATA2(IANT,14) = JSEED
          RDATA2(IANT,15) = N0
          RDATA2(IANT,16) = CXT
          RDATA2(IANT,17) = CYT
          RDATA2(IANT,18) = VERSON
```

Table 16. ACIRF Fortran code (Continued).

```
         RDATA2(IANT,21) = NUMANT
         RDATA2(IANT,22) = FREQC
         RDATA2(IANT,23) = IANT
         RDATA2(IANT,24) = BWU(IANT)
         RDATA2(IANT,25) = BWV(IANT)
         RDATA2(IANT,26) = ROT(IANT)
         RDATA2(IANT,27) = UPOS(IANT)
         RDATA2(IANT,28) = VPOS(IANT)
         RDATA2(IANT,29) = ELV(IANT)
         RDATA2(IANT,30) = AZM(IANT)
         RDATA2(IANT,31) = PA(IANT)
         RDATA2(IANT,32) = SLOSDB(IANT)
C
C  OPEN BINARY FILES - ONE FOR EACH ANTENNA
C
         IUNIT = IANT - 1 + IOFILE
         NAME = 'ACIRF.AN'//CHAR(48+IANT)
         OPEN(UNIT=IUNIT,FILE=NAME,STATUS='NEW',FORM='UNFORMATTED')
         WRITE(IUNIT) MDENT,IDENT(1:MDENT)
         WRITE(IUNIT) NDATA1,(RDATA1(II),II=1,NDATA1)
         WRITE(IUNIT) NDATA2,(RDATA2(IANT,II),II=1,NDATA2)
  120 CONTINUE
C
C******  CHANNEL SIMULATOR
C
      CALL CHANL1
C
C******  MEASURE IMPULSE RESPONSE FUNCTION STATISTICS
C
      CALL MEASR1
      IJSEED = IOR(ISHFT(ISEED,16),JSEED)
      WRITE(IWRITE,2100)IJSEED
 2100 FORMAT(/2X,'FINAL RANDOM NUMBER SEED = ',I12)
      CLOSE(UNIT=IWRITE)
      STOP '  ACIRF COMPLETED'
      END
```

Table 17. CHANL1 Fortran code.

```
      SUBROUTINE CHANL1
C
C********************* VERSION 3.3 *** 08 JAN 1990 *****************************
C
C  THIS SUBROUTINE CALCULATES IMPULSE RESPONSE FUNCTION FOR WITTWER'S GENERAL
C  MODEL WHICH INCLUDES TIME-SPACE CORRELATION
C
C**********************  SET ARRAY SIZES  *************************************
C
      INCLUDE 'ACIRF.SIZ'
C
C*******************************************************************************
C
      LOGICAL RESET
      COMMON/ANTENA/NUMANT,XPOS(MTENNA),YPOS(MTENNA)
      COMMON/ANTNUM/IANT
      COMMON/CHANEL/DLX,DLY,F0,CXT,CYT
      COMMON/GRDSIZ/DELTAU,DELTKX,DELTKY,TAU0DF
      COMMON/KXYPWR/CUMDKX,CUMDKY,COSTH,SINTH,DELTKM,DELTKN,DLM,DLN,
     1  MIDX,MIDY,AKMMAX,AKNMAX
      COMMON/IMPULS/HA(MTIMES,MDELAY),HB(MTIMES),HC(MDELAY)
      COMMON/INOUT/IREAD,IWRITE,IOFILE
      COMMON/PROFIL/BEAM1(MTENNA),BEAM2(MTENNA),BEAM3(MTENNA),
     1  PNTAKX(MTENNA),PNTAKY(MTENNA)
      COMMON/POWERS/G(MDELAY,MTENNA),GRDPOW(3,MTENNA)
      COMMON/RANSED/ISEED,JSEED
      COMMON/REALZN/NKX,NKY,NDELAY,NTIMES,N0,NFMAX
      COMPLEX AWGN,HK(MKY,MKX),HA,HB,HC,YPHSR(MKY)
      COMPLEX PHSR,DPHSR
      DIMENSION AMP(MKY,MKX),JDEL(MKY,MKX)
      DIMENSION RDUM1(NDATA1),RDUM2(NDATA2),DAKY(MKY)
      CHARACTER IDENT*80
      PARAMETER (PI=3.141592654,TWOPI=2.0*PI)
      PARAMETER (SMALL=1.0E-10)
      EQUIVALENCE (HK(1,1),HA(1,1))
C
C******  PRECOMPUTE COEFFICIENTS OF GENERALIZED POWER SPECTRUM
C
      AKANGL = SQRT(DLX**4*DLY**4/(8.0*(DLY**4 + DLX**4)))/
     1  (TWOPI*F0*DELTAU)
      MIDX = NKX/2 + 1
      MIDY = NKY/2 + 1
      DO 10 IANT=1,NUMANT
          GRDPOW(1,IANT) = 0.0
   10 CONTINUE
C
C******  CALCULATE AMPLITUDES FOR ANGULAR GRID AT ZERO FREQUENCY
C
C  CALCULATE ANGULAR ENERGY IN DIAGONALIZED KM-KN COORDINATE SYSTEM
C     STRICTLY A BRUTE FORCE APPROACH FOR ASSIGNING ENERGY TO THE
C     KX-KY GRID FROM THE DIAGONAL KM-KN GRID
```

Table 17. CHANL1 Fortran code (Continued).

```
C
C  ROTATION ANGLE FROM KX-XY TO KM-KN
C
      BOT = DLX*(1.0-CYT**2)/DLY-DLY*(1.0-CXT**2)/DLX
      IF(ABS(BOT) .GT. 1.0E-07)THEN
          THETA = 0.5*ATAN(2.0*CXT*CYT/BOT)
      ELSE
          THETA = PI/4.0
      END IF
      COSTH = COS(THETA)
      SINTH = SIN(THETA)
C
C  KM-KN DECORRELATION DISTANCES, CORRELATION COEFFICIENTS, AND GRID SIZES
C
      DLM = 1.0/SQRT((COSTH/DLX)**2+(SINTH/DLY)**2)
      DLN = 1.0/SQRT((SINTH/DLX)**2+(COSTH/DLY)**2)
      CMT = DLM*(CXT*COSTH/DLX+CYT*SINTH/DLY)
      CNT = DLN*(CYT*COSTH/DLY-CXT*SINTH/DLX)
      DELTKM = 0.099/SQRT((COSTH/DELTKX)**2+(SINTH/DELTKY)**2)
      DELTKN = 0.099/SQRT((SINTH/DELTKX)**2+(COSTH/DELTKY)**2)
      DLM = DLM/SQRT(1.0-CMT**2)
      DLN = DLN/SQRT(1.0-CNT**2)
C
C  KM-KN GRID LIMITS - CONTAIN 99.9 PERCENT OF ENERGY IN ANGULAR-DOPPLER GRID
C
      AKMMAX = 5.1790/DLM
      AKNMAX = 5.1790/DLN
C
C******  CALCULATE KX-KY ANGULAR GRID AMPLITUDE = SQRT(ENERGY)
C
C  ZERO CUMULATIVE DOPPLER KX-KY OFFSETS
C
      CUMDKX = 0.0
      CUMDKY = 0.0
C
C  AMPLITUDE ARRAY AT ZERO DOPPLER FREQUENCY
C
      CALL FILAMP(AMP,1,NKX,1,NKY)
C
C******  START LOOP OVER POSITIVE DOPPLER FREQUENCIES
C        ADD POWER OF ZERO DOPPLER FREQUENCY BIN TO ADJACENT + AND - BINS
C
      PITOF1 = 0.0
      DO 120 NF=2,NFMAX
C
C  START WITH POSITIVE FREQUENCY
C
          PITOF2 = PI*TAUODF*(FLOAT(NF)-0.5)
          AMPDOP = SQRT(0.5*(ERFC(PITOF1)-ERFC(PITOF2)))
          PITOF1 = PITOF2
C
C  FIND DOPPLER KX-KY OFFSETS
C    IF OFFSET IS LARGER THAN KX-KY GRID CELL, THEN OFFSET
C    AMP(KX,KY) ARRAY AND FILL IN NEW VALUES
C
```

Table 17. CHANL1 Fortran code (Continued).

```
      AKXDIS = CXT*TWOPI*TAUODF*(NF-1)/DLX - CUMDKX
      J = INT(AKXDIS/DELTKX)
      IF(J .NE. 0)THEN
C
C  OFFSET AMPLITUDE ARRAY IN POSITIVE KX DIRECTION
C
          IF(J .GT. 0)THEN
              KEND = NKX - J
              MM = KEND + 1
              DO 25 KK=1,KEND
                  MM = MM - 1
                  JJ = MM + J
                  DO 20  LL=1,NKY
                      AMP(LL,JJ)  = AMP(LL,MM)
   20             CONTINUE
   25         CONTINUE
              KSTRT = 1
              KEND  = J
          END IF
C
C  OFFSET AMPLITUDE ARRAY IN NEGATIVE KX DIRECTION
C
          IF(J .LT. 0)THEN
              KEND = NKX + J
              DO 35 KK=1,KEND
                  MM = KK - J
                  DO 30 LL=1,NKY
                      AMP(LL,KK)  = AMP(LL,MM)
   30             CONTINUE
   35         CONTINUE
              KSTRT = NKX + J + 1
              KEND  = NKX
          END IF
          AKX = J*DELTKX
          CUMDKX = CUMDKX + AKX
          AKXDIS = AKXDIS - AKX
C
C  FILL IN NEW KX AMPLITUDES
C
          CALL FILAMP(AMP,KSTRT,KEND,1,NKY)
      END IF
      AKYDIS = CYT*TWOPI*TAUODF*(NF-1)/DLY - CUMDKY
      J = INT(AKYDIS/DELTKY)
      IF(J .NE. 0)THEN
C
C  OFFSET AMPLITUDE ARRAY IN POSITIVE KY DIRECTION
C
          IF(J .GT. 0)THEN
              LEND = NKY - J
              MM = LEND + 1
              DO 45 LL=1,LEND
                  MM = MM - 1
                  JJ = MM + J
                  DO 40 KK=1,NKX
                      AMP(JJ,KK)  = AMP(MM,KK)
  40              CONTINUE
```

Table 17. CHANL1 Fortran code (Continued).

```
 45              CONTINUE
                 LSTRT = 1
                 LEND  = J
             END IF
C
C  OFFSET AMPLITUDE ARRAY IN NEGATIVE KY DIRECTION
C
             IF(J .LT. 0)THEN
                 LEND = NKY + J
                 DO 55 LL=1,LEND
                     MM = LL - J
                     DO 50 KK=1,NKX
                         AMP(LL,KK) = AMP(MM,KK)
 50                  CONTINUE
 55              CONTINUE
                 LSTRT = NKY + J + 1
                 LEND = NKY
             END IF
             AKY = J*DELTKY
             CUMDKY = CUMDKY + AKY
             AKYDIS = AKYDIS - AKY
C
C  FILL IN NEW KY AMPLITUDES
C
             CALL FILAMP(AMP,1,NKX,LSTRT,LEND)
         END IF
C
C  CALCULATION FOR POSITIVE DOPPLER FREQUENCY THEN CORRESPONDING
C    NEGATIVE DOPPLER FREQUENCY
C
         FSIGN = 1.0
         DO 110 ISIGN=1,2
C
C******  ASSIGN KX-KY GRID CELLS TO DELAY BINS BASED ON DELAY-ANGLE
C          DELTA FUNCTION
C
             AKM = (FLOAT(1-MIDX)-0.5)*DELTKX + AKXDIS
             DO 65 KK=1,NKX
                 AKN = (FLOAT(1-MIDY)-0.5)*DELTKY + AKYDIS
                 DO 60 LL=1,NKY
                     JDEL(LL,KK) = 0
                     IF(AMP(LL,KK) .GT. SMALL)THEN
C
C  RANDOMLY WIGGLE KX-KY CELL LOCATIONS TO INSURE CORRECT
C    AVERAGE ENERGY IN DELAY BINS
C
                         AKX = AKM + RANDOM(ISEED,JSEED)*DELTKX
                         AKY = AKN + RANDOM(ISEED,JSEED)*DELTKY
C
C  RECORD THE DELAY INDEX FOR EACH KX-KY CELL AND SET RANDOM FOURIER
C  COEFFICIENT IF KX-KY GRID CELL IS WITHIN DELAY GRID
C
                        J = INT(AKANGL*(AKX**2 + AKY**2))
                        IF(J .LT. NDELAY)THEN
                            JDEL(LL,KK) = J + 1
```

Table 17. CHANL1 Fortran code (Continued).

```
                         HK(LL,KK) = AMPDOP*AMP(LL,KK)*
     1                     AWGN(ISEED,JSEED)
                      END IF
                   END IF
                   AKN = AKN + DELTKY
   60           CONTINUE
                AKM = AKM + DELTKX
   65        CONTINUE
C
C******  START LOOP OVER ANTENNAS
C
             DO 100 IANT=1,NUMANT

C
C  ZERO DELAY ARRAY AND POWER ACCUMULATOR
C
                DO 70 J=1,NDELAY
                   HC(J) = (0.0,0.0)
   70           CONTINUE
                AVEPOW = 0.0
C
C  SET UP DFT PHASOR ARRAY AND NORMALIZE IT FOR DELAY INTEGRATION
C  (/DELTAU); SET KY ARRAY WITH POINTING ANGLE OFFSET;  AND SET INITIAL
C  KX PHASOR VARIABLES.  THIS ALL SPEEDS UP DFT'S BELOW.
C
                DAKY(1) = FSIGN*((1-MIDY)*DELTKY + AKYDIS) -
     1            PNTAKY(IANT)
                THETA = DAKY(1)*YPOS(IANT)
                YPHSR(1) = CMPLX(COS(THETA),SIN(THETA))/DELTAU
                AKY = FSIGN*DELTKY
                THETA = AKY*YPOS(IANT)
                DPHSR = CMPLX(COS(THETA),SIN(THETA))
                DO 80 LL=2,NKY
                    DAKY(LL) = DAKY(LL-1) + AKY
                    YPHSR(LL) = YPHSR(LL-1)*DPHSR
   80           CONTINUE
                AKX = FSIGN*(AKXDIS-MIDX*DELTKX) -
     1            PNTAKX(IANT)
                THETA = (AKX + FSIGN*DELTKX)*XPOS(IANT)
                PHSR = CMPLX(COS(THETA),SIN(THETA))
                THETA = FSIGN*DELTKX*XPOS(IANT)
                DPHSR = CMPLX(COS(THETA),SIN(THETA))
C
C  SET BEAM PROFILE CONSTANTS TO SPEED UP KX-KY LOOP BELOW
C
                BMXX = BEAM1(IANT)
                BMYY = BEAM2(IANT)
                BMXY = BEAM3(IANT)
C
C  START LOOP OVER ANGULAR GRID - PERFORM KX TO X AND KY TO Y DFTS
C
                DO 95 KK=1,NKX
                    AKXX = AKX + FSIGN*KK*DELTKX
                    AKXY = AKXX*BMXY
                    AKXX = AKXX*AKXX*BMXX
                    DO 90 LL=1,NKY
```

Table 17. CHANL1 Fortran code (Continued).

```
C
C  SELECT ANGULAR SAMPLES THAT BELONG TO EACH DELAY BIN AND ACCUMULATE
C  GRID ENERGY
C
                          J = JDEL(LL,KK)
                          IF(J .GT. 0)THEN
                              CGAIN = EXP(-0.5*(DAKY(LL)*(DAKY(LL)*
     1                          BMYY + AKXY) + AKXX))
                              HC(J) = HC(J) +
     1                          HK(LL,KK)*PHSR*YPHSR(LL)*CGAIN
                              AVEPOW = AVEPOW + (CGAIN*AMP(LL,KK))**2
                          END IF
   90                 CONTINUE
                      PHSR = PHSR*DPHSR
   95             CONTINUE
C
C  ACCUMULATE KX-KY GRID POWER
C
                  GRDPOW(1,IANT) = GRDPOW(1,IANT) + AVEPOW*AMPDOP**2
C
C  WRITE DELAY-DOPPLER IMPULSE RESPONSE FUNCTIONS INTO BINARY FILES
C
                  IUNIT = IANT - 1 + IOFILE
                  WRITE(IUNIT) (HC(J),J=1,NDELAY)
                  IF(MOD(NF,NO) .EQ. 0 .AND. ISIGN .EQ. 2)THEN
                      NUMFRQ = 2*NF
                      WRITE(*,3000)NUMFRQ,IANT
 3000                 FORMAT(2X,'FREQ ',I5,' FOR ANTENNA ',I2,
     1                  ' COMPLETE')
                  END IF
C
C  GO TO NEXT ANTENNA
C
  100         CONTINUE
              FSIGN = -FSIGN
C
C  DONE WITH POSITIVE DOPPLER FREQUENCY, DO CORRESPONDING NEGATIVE FREQUENCY
C
  110     CONTINUE
C
C  GO TO NEXT POSITIVE DOPPLER FREQUENCY
C
  120 CONTINUE
C
C   RESET WRITER SUBROUTINE
C
      RESET = .TRUE.
      CALL WRITER(RESET,NDELAY,NTIMES,IANT,IUNIT,HC)
C
C******  GENERATE TIME SEQUENCES BY PERFORMING FAST FOURIER TRANSFORM
C           FROM DOPPLER DOMAIN TO TIME DOMAIN
C
      IFFT = 1
      NN = INT(ALOG(FLOAT(NTIMES))/ALOG(1.999))
      LEND = 1 + NTIMES/2
```

Table 17. CHANL1 Fortran code (Continued).

```
C
C  LOOP OVER ANTENNAS
C
      DO 270 IANT=1,NUMANT
          IUNIT = IANT - 1 + IOFILE
          REWIND(IUNIT)
          READ (IUNIT) NDUM1,IDENT(1:NDUM1)
          READ (IUNIT) NDUM1,(RDUM1(I),I=1,NDUM1)
          READ (IUNIT) NDUM2,(RDUM2(I),I=1,NDUM2)
C
C  READ DELAY-DOPPLER IMPULSE RESPONSE FUNCTIONS
C    POSITIVE DOPPLER FREQUENCY SAMPLES GO INTO BEGINNING OF ARRAY
C    NEGATIVE DOPPLER FREQUENCY SAMPLES GO INTO END OF ARRAY
C
          JJ = NTIMES
          DO 210 NT=2,NFMAX
              READ(IUNIT)(HA(NT,J),J=1,NDELAY)
              READ(IUNIT)(HA(JJ,J),J=1,NDELAY)
              JJ = JJ - 1
 210      CONTINUE
C
C  ZERO ELEMENTS FOR NON-CALCULATED DOPPLER FREQUENCIES
C
          LL = NFMAX + 1
          DO 225 NT=LL,LEND
              DO 220 J=1 NDELAY
                  HA(NT,J) = (0.0,0.0)
                  HA(JJ,J) = (0.0,0.0)
 220          CONTINUE
              JJ = JJ-1
 225      CONTINUE
C
C  PREPARE FILE TO READ IN DELAY-TIME IMPULSE RESPONSE FUNCTION
C
          REWIND(IUNIT)
          READ (IUNIT) NDUM1,IDENT(1:NDUM1)
          READ (IUNIT) NDUM1,(RDUM1(I),I=1,NDUM1)
          READ (IUNIT) NDUM2,(RDUM2(I),I=1,NDUM2)
          DO 250 J=1,NDELAY
              DO 230 KK=2,NTIMES
                  HB(KK) = HA(KK,J)
 230          CONTINUE
C
C  ZERO DC FREQUENCY TERM, ENERGY HAS ALREADY BEEN DIVIDED BETWEEN
C    HC(2)      - THE SMALLEST POSITIVE FREQUENCY AND
C    HC(NTIMES) - THE LARGEST NEGATIVE FREQUENCY
C
              HB(1) = (0.0,0.0)
C
C  USE FFT TO CALCULATE TIME ARRAY AT GIVEN DELAY FOR CURRENT ANTENNA
C
              CALL FFT(HB,NN,NTIMES,IFFT)
              DO 240 KK=1,NTIMES
                  HA(KK,J) = HB(KK)
 240          CONTINUE
```

Table 17. CHANL1 Fortran code (Continued).

```
C
C  GO TO NEXT DELAY
C
 250      CONTINUE
C
C  READ TIME SEQUENCE INTO FILE
C
          DO 265 KK=1,NTIMES
              DO 260 J=1,NDELAY
                  HC(J) = HA(KK,J)
  260         CONTINUE
              CALL WRITER(RESET,NDELAY,NTIMES,IANT,IUNIT,HC)
  265     CONTINUE
          WRITE(*,3001)IANT
 3001     FORMAT(2X,'ANTENNA ',I2,' COMPLETE')
C
C  NEXT ANTENNA
C
  270 CONTINUE
      RETURN
      END
```

Table 18. FILAMP Fortran code.

```
      SUBROUTINE FILAMP(AMP,KSTRT,KEND,LSTRT,LEND)
C
C********************* VERSION 1.3 *** 08 JAN 1990 ****************************
C
C  GENERATE GPSD AMPLITUDE ON KM-KN GRID AND ASSIGN TO KX-XY GRID CELL
C
C******************************************************************************
C
      INCLUDE 'ACIRF.SIZ'
      PARAMETER (SMALL=0.00001)
      COMMON/GRDSIZ/DELTAU,DELTKX,DELTKY,TAU0DF
      COMMON/KXYPWR/CUMDKX,CUMDKY,COSTH,SINTH,DELTKM,DELTKN,DLM,DLN,
     1  MIDX,MIDY,AKMMAX,AKNMAX
      DIMENSION AMP(MKY,MKX)
C
C****** ZERO AMPLITUDE OF NEW GRID CELLS
C
      DO 15 KK=KSTRT,KEND
          DO 10 LL=LSTRT,LEND
              AMP(LL,KK) = 0.0
   10     CONTINUE
   15 CONTINUE
C
C****** FIND LIMITS OF KX-KY REGION ON KM-KN GRID
C
      AKX1 = (FLOAT(KSTRT-MIDX) - 0.5)*DELTKX - CUMDKX
      AKX2 = (FLOAT(KEND-MIDX)  + 0.5)*DELTKX - CUMDKX
      AKY1 = (FLOAT(LSTRT-MIDY) - 0.5)*DELTKY - CUMDKY
      AKY2 = (FLOAT(LEND-MIDY)  + 0.5)*DELTKY - CUMDKY
C
C  CALCULATE KM,KM COORDINATES OF KX-KY REGION
C
      AKM1 = AKX1*COSTH + AKY1*SINTH
      AKM2 = AKX1*COSTH + AKY2*SINTH
      AKM3 = AKX2*COSTH + AKY2*SINTH
      AKM4 = AKX2*COSTH + AKY1*SINTH
      AKN1 = AKY1*COSTH - AKX1*SINTH
      AKN2 = AKY2*COSTH - AKX1*SINTH
      AKN3 = AKY2*COSTH - AKX2*SINTH
      AKN4 = AKY1*COSTH - AKX2*SINTH
      IF(AKM2 .LT. AKM1)THEN
C
C  INSURE THAT AKM1 IS THE MINIMUM KM COORDINATE
C    IF  AKM2 < AKM1 THEN RENAME KM,KN COORDINATES
C
          AKM  = AKM1
          AKN  = AKN1
          AKM1 = AKM2
          AKN1 = AKN2
          AKM2 = AKM3
          AKN2 = AKN3
          AKM3 = AKM4
          AKN3 = AKN4
          AKM4 = AKM
          AKN4 = AKN
```

Table 18. FILAMP Fortran code (Continued).

```
      END IF
C
C******  IF KX-KY REGION LIES OUTSIDE OF THE KM-KN REGION THAT CONTAINS
C          SIGNAL ENERGY, THEN RETURN.  99.9 PERCENT OF SIGNAL ENERGY LIES
C          WITHIN THE REGION -KMMAX TO +KMMAX AND -KNMAX TO +KNMAX
C
      IF(AKN2 .LT. -AKNMAX .OR. AKN4 .GT. AKNMAX)RETURN
      IF(AKM3 .LT. -AKMMAX .OR. AKM1 .GT. AKMMAX)RETURN
C
C******  CALCULATE LIMITS ON KM AND KN IMPOSED BY KX-KY REGION AND AKNMAX
C          DRAW A LINE ACROSS THE KM-KN REGION THAT BEST DESCRIBES THE
C          LOCATION, IN THE KM-KN PLANE, OF THE SIGNAL ENERGY WITHIN
C          THE KX-KY REGION.  LIMIT END POINTS OF LINE TO BE WITHIN
C          -KNMAX TO +KNMAX (THE KM LIMITS WILL BE IMPOSED LATER).
C
      SMN = SMALL*DELTKN
      AKX2 = AKM3
      AKY2 = AKN3
      IF(AKN1 .LE. AKNMAX .AND. AKN1 .GE. -AKNMAX)THEN
C
C  CASE 1: KN1 .LE. KNMAX AND KN1 .GE. -KNMAX
C
        AKX1 = AKM1
        AKY1 = AKN1
        IF(AKN3 .GT. AKNMAX)THEN
C
C  CASE 1A:  KN3 .GT. KNMAX
C
          AKX2 = AKM4 + (AKNMAX-AKN4)*(AKM3-AKM4)/(AKN3-AKN4)
          AKY2 = AKNMAX
        ELSE IF(AKN3 .LT. -AKNMAX)THEN
C
C  CASE 1B:  KN3 .LT. -KNMAX
C
          AKX2 = AKM2 - (AKNMAX+AKN2)*(AKM3-AKM2)/(AKN3-AKN2)
          AKY2 = -AKNMAX
        END IF
      ELSE
C
C  CASE 2:  KN1 .GT. KNMAX .OR. KN1 .LT. -KNMAX
C
        IF(AKN1 .GE. (AKNMAX-SMN))THEN
C
C  CASE 2A:  KN1 .GE. KNMAX
C
          AKX1 = AKM1 - (AKNMAX-AKN1)*(AKM4-AKM1)/(AKN1-AKN4+SMN)
          AKY1 = AKNMAX
C
C  CASE 2A-1:  KN1 .GT. KNMAX .AND. KN3 .GT. KNMAX
C
          IF(AKN3 .GT. AKNMAX)THEN
            AKX2 = AKM4 + (AKNMAX-AKN4)*(AKM3-AKM4)/(AKN3-AKN4+SMN)
            AKY2 = AKNMAX
```

Table 18. FILAMP Fortran code (Continued).

```
         ELSE IF(AKN3 .LT. -AKNMAX)THEN
C
C  CASE 2A-2:  KN1 .GT. KNMAX .AND. KN3 .LT. -KNMAX
C
           AKX2 = AKM2 - (AKNMAX+AKN2)*(AKM3-AKM2)/(AKN3-AKN2)
           AKY2 = -AKNMAX
         END IF
       ELSE
C
C  CASE 2B:  KN1 .LT. -KNMAX
C
         AKX1 = AKM1 - (AKNMAX+AKN1)*(AKM2-AKM1)/(AKN2-AKN1+SMN)
         AKY1 = -AKNMAX
         IF(AKN3 .LT. -AKNMAX)THEN
C
C  CASE 2B-1:  KN1 .LT. -KNMAX .AND. KN3 .LT. -KNMAX
C
           AKX2 = AKM3 - (AKNMAX+AKN3)*(AKM2-AKM3)/(AKN2-AKN3+SMN)
           AKY2 = -AKNMAX
         ELSE IF(AKN3 .GT. AKNMAX)THEN
C
C  CASE 2B-2:  KN1 .LT. -KNMAX .AND. KN3 .GT. KNMAX
C
           AKX2 = AKM4 + (AKNMAX-AKN4)*(AKM3-AKM4)/(AKN3-AKN4)
           AKY2 = AKNMAX
         END IF
       END IF
     END IF
C
C******  CALCULATE ENERGY ON KM-KN GRID AND ASSIGN TO KX-KY GRID CELLS
C
C  DEFINE STARTING POINT OF LINE AND ITS SLOPE
C
     SLOPE = (AKY2-AKY1)/(AKX2-AKX1)
     AKM0  = AKX1
C
C  RESET END POINTS OF LINE IF OUTSIDE THE LIMITS -KMMAX TO +KMMAX
C
     IF(AKX1 .LT. -AKMMAX)AKX1 = -AKMMAX
     IF(AKX2 .GT.  AKMMAX)AKX2 =  AKMMAX
C
C  CALCULATE NO OF KM CELLS FROM KM=0 TO END POINTS OF LINE AND
C            NO OF KN CELLS FROM KN=0 TO KNMAX
C
     KMMIN = INT(SIGN(ABS(AKX1/DELTKM) + 0.5,AKX1))
     KMMAX = INT(SIGN(ABS(AKX2/DELTKM) + 0.5,AKX2))
     IF(KMMAX .LT. KMMIN)RETURN
     KNMAX = INT(0.5 + AKNMAX/DELTKN)
C
C  DEFINE CONSTANTS AND CENTER OF KX-KY REGION
C
     AKXMID = (FLOAT(MIDX) + 0.5)*DELTKX + CUMDKX
     AKYMID = (FLOAT(MIDY) + 0.5)*DELTKY + CUMDKY
     DLM2 = DLM/2.0
     DLN2 = DLN/2.0
C
```

Table 18. FILAMP Fortran code (Continued).

```
C******  LOOP OVER KM FOLLOWING LINE FROM (KX1,KY1) TO (KX2,KY2)
C
      DO 40 KM=KMMIN,KMMAX
          AKM = KM*DELTKM
          AKM1 = (AKM - 0.5*DELTKM)*DLM2
          AKM2 = AKM1 + DELTKM*DLM2
          EKM = 0.5*(ERFC(AKM1)-ERFC(AKM2))
          AKN = AKY1 + (AKM-AKM0)*SLOPE
          KNLINE = INT(SIGN(ABS(AKN/DELTKN) + 0.5,AKN))
C
C******  LOOP OVER KN
C
C  START AT LINE.  FIRST INCREASE KN VALUES UNTIL OUT OF KX-KY REGION.
C  RETURN TO LINE AND DECREASE KN VALUES UNTIL AGAIN OUT OF KX-KY REGION.
C
          DO 30 IUPDN=1,2
              IF(IUPDN .EQ. 1)THEN
                  KNSTRT = KNLINE
                  KNEND  = KNMAX
                  KNINC  = +1
              ELSE
                  KNSTRT = KNLINE - 1
                  KNEND  = -KNMAX
                  KNINC  = -1
              END IF
              DO 20 KN=KNSTRT,KNEND,KNINC
                  AKN = KN*DELTKN
                  KK = INT((AKM*COSTH-AKN*SINTH+AKXMID)/DELTKX)
                  LL = INT((AKM*SINTH+AKN*COSTH+AKYMID)/DELTKY)
C
C  TEST IF INSIDE OF KX-KY REGION
C
                  IF(KK.LT.KSTRT .OR. KK.GT.KEND .OR.
     1               LL.LT.LSTRT .OR. LL.GT.LEND)THEN
                      IF(KN .EQ. KNLINE)GO TO 20
                      GO TO 30
                  END IF
C
C  IF IN KX-KY REGION, ADD SOME ENERGY TO THE (LL,KK) CELL
C
                  AKN1 = (AKN - 0.5*DELTKN)*DLN2
                  AKN2 = AKN1 + DELTKN*DLN2
                  EKN = 0.5*(ERFC(AKN1) - ERFC(AKN2))
                  AMP(LL,KK) = AMP(LL,KK) + EKM*EKN
   20         CONTINUE
   30     CONTINUE
   40 CONTINUE
C
C  FINISH UP WITH CALCULATION OF AMPLITUDE
C
      DO 55 KK=KSTRT,KEND
          DO 50 LL=LSTRT,LEND
              AMP(LL,KK) = SQRT(AMP(LL,KK))
   50     CONTINUE
   55 CONTINUE
      RETURN
      END
```

Table 19. MEASR1 Fortran code.

```
      SUBROUTINE MEASR1
C
C********************* VERSION 1.13 *** 25 APR 1991 ***************************
C
C  THIS SUBROUTINE MEASURES AND WRITES THE STATISTICS
C  OF THE IMPULSE RESPONSE FUNCTION REALIZATIONS GENERATED BY ACIRF
C
C**********************  SET ARRAY SIZES  ************************************
C
      INCLUDE 'ACIRF.SIZ'
C
C******************************************************************************
C
      CHARACTER*10 RHOAMP(MTENNA),RHOPHS(MTENNA)
      CHARACTER*80 IDENT
      LOGICAL RESET
      COMMON/ANTENA/NUMANT,XPOS(MTENNA),YPOS(MTENNA)
      COMMON/ENSMBL/CROSS(MTENNA,MTENNA),PA(MTENNA),SLOSDB(MTENNA),
     1  FA(MTENNA),TAUA(MTENNA)
      COMMON/IMPULS/HA(MTIMES,MDELAY),HB(MTIMES),HC(MDELAY)
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/POWERS/G(MDELAY,MTENNA),GRDPOW(3,MTENNA)
      DIMENSION AMPAVE(MDELAY+1,4),S4(MDELAY+1),CHIBAR(MDELAY+1,2)
      DIMENSION P0(MTENNA),TEMP(MTENNA)
      DIMENSION HEADER(NDATA1)
      COMPLEX HA,HB,HC,HD,CROSS
      PARAMETER (PI=3.141592654,TWOPI=2.0*PI,EULER=0.5772157)
      PARAMETER (X1NORM=-EULER/2.0,X2NORM=PI**2/24.0+EULER**2/4.0)
      DATA RHOAMP/'  AMP(N-1)','  AMP(N-2)','  AMP(N-3)','  AMP(N-4)',
     1            '  AMP(N-5)','  AMP(N-6)','  AMP(N-7)','  AMP(N-8)'/
      DATA RHOPHS/'  PHS(N-1)','  PHS(N-2)','  PHS(N-3)','  PHS(N-4)',
     1            '  PHS(N-5)','  PHS(N-6)','  PHS(N-7)','  PHS(N-8)'/
C
C******  READ REALIZATIONS AND COLLECT STATISTICS
C
      DO 100 IANT=1,NUMANT
      IUNIT = IANT - 1 + IOFILE
      WRITE(IWRITE,2000)IANT
 2000 FORMAT(1H1,1X,'MEASURED PARAMETERS FOR REALIZATION/',
     1  'ANTENNA ',I1)
C
C  READ HEADER RECORDS
C
      REWIND IUNIT
      READ(IUNIT) MDENT,IDENT(1:MIN0(MDENT,80))
      READ(IUNIT) MDATA1,(HEADER(II),II=1,MIN0(MDATA1,NDATA1))
      NTIMES = HEADER(14) + 0.01
      DELTAT = HEADER(15)
      NO     = HEADER(16) + 0.01
      NDELAY = HEADER(20) + 0.01
      JT     = NDELAY + 1
      TAUMIN = HEADER(21)
      DELTAU = HEADER(22)
      RESET  = .TRUE.
      CALL READER(RESET,NDELAY,NTIMES,IANT,IUNIT,HC)
```

Table 19. MEASR1 Fortran code (Continued).

```
C
C  INITIALIZE MEASUREMENTS
C
      DO 20 J=1,JT
          CHIBAR(J,1) = 0.0
          CHIBAR(J,2) = 0.0
          DO 10 M=1,4
              AMPAVE(J,M) = 0.0
   10     CONTINUE
   20 CONTINUE
      AVENRG = 0.0
      AVETAU = 0.0
      SIGTAU = 0.0
      PO(IANT) = 0.0
      DO 40 KK=1,NTIMES
C
C  READ IMPULSE RESPONSE FUNCTION
C
          CALL READER(RESET,NDELAY,NTIMES,IANT,IUNIT,HC)
          HA(KK,IANT) = (0.0,0.0)
          HD = (0.0,0.0)
C
C  COLLECT STATISTICS VERSUS DELAY
C
          DO 30 J=1,NDELAY
              AMP = CABS(HC(J))*DELTAU
              IF(AMP .GT. 0.0)THEN
                  POW = AMP**2
                  HD = HD + HC(J)*DELTAU
                  HA(KK,IANT) = HA(KK,IANT) + HC(J)*DELTAU
                  AMPAVE(J,1) = AMPAVE(J,1) + AMP
                  AMPAVE(J,2) = AMPAVE(J,2) + POW
                  AMPAVE(J,3) = AMPAVE(J,3) + AMP**3
                  AMPAVE(J,4) = AMPAVE(J,4) + POW**2
                  AMP = ALOG(AMP)
                  CHIBAR(J,1) = CHIBAR(J,1) + AMP
                  CHIBAR(J,2) = CHIBAR(J,2) + AMP**2
                  TAU = TAUMIN + (J-1)*DELTAU
                  AVENRG = AVENRG + POW
                  AVETAU = AVETAU + POW*TAU
                  SIGTAU = SIGTAU + POW*TAU**2
              END IF
   30     CONTINUE
          HB(KK) = HA(KK,IANT)
          PO(IANT) = PO(IANT) + CABS(HA(KK,IANT))**2
          AMP = CABS(HD)
          AMPAVE(JT,1) = AMPAVE(JT,1) + AMP
          AMPAVE(JT,2) = AMPAVE(JT,2) + AMP**2
          AMPAVE(JT,3) = AMPAVE(JT,3) + AMP**3
          AMPAVE(JT,4) = AMPAVE(JT,4) + AMP**4
          AMP = ALOG(AMP)
          CHIBAR(JT,1) = CHIBAR(JT,1) + AMP
          CHIBAR(JT,2) = CHIBAR(JT,2) + AMP**2
   40 CONTINUE
```

Table 19. MEASR1 Fortran code (Continued).

```
C
C******  MEASURED DECORRELATION TIME
C
      CALL RHO(HB,NTIMES,XTAU0)
      TAU0 = 1.0E30
      IF(XTAU0 .GT. 0.0)TAU0 = XTAU0*DELTAT
C
C******  MEASURED MOMENTS OF AMPLITUDE VERSUS DELAY
C
      DO 60 J=1,JT
          DO 50 M=1,4
              AMPAVE(J,M) = AMPAVE(J,M)/NTIMES
   50     CONTINUE
          CHIBAR(J,1) = CHIBAR(J,1)/NTIMES
          CHIBAR(J,2) = CHIBAR(J,2)/NTIMES
   60 CONTINUE
      DO 70 J=1,NDELAY
          PJ = G(J,IANT)
          IF(AMPAVE(J,2)**2 .GT. 0.0 .AND. PJ**2 .GT. 0.0)THEN
              S4(J) = (AMPAVE(J,4)-AMPAVE(J,2)**2)/AMPAVE(J,2)**2
              IF(S4(J) .GT. 0.0)THEN
                  S4(J) = SQRT(S4(J))
              ELSE
                  S4(J) = 0.0
              END IF
              AMPAVE(J,1) = AMPAVE(J,1)/(0.5*SQRT(PI))/SQRT(PJ)
              AMPAVE(J,2) = AMPAVE(J,2)/PJ
              AMPAVE(J,3) = AMPAVE(J,3)/(0.75*SQRT(PI))/SQRT(PJ)**3
              AMPAVE(J,4) = AMPAVE(J,4)/2.0/PJ**2
              AMP =0.5*ALOG(PJ)
              CHIBAR(J,2) = (CHIBAR(J,2)-2.0*CHIBAR(J,1)*AMP+AMP**2)/
     1            X2NORM
              CHIBAR(J,1) = (CHIBAR(J,1)-AMP)/X1NORM
          ELSE
              S4(J) = 0.0
              AMPAVE(J,1) = 0.0
              AMPAVE(J,2) = 0.0
              AMPAVE(J,3) = 0.0
              AMPAVE(J,4) = 0.0
              CHIBAR(J,1) = 0.0
              CHIBAR(J,2) = 0.0
          END IF
   70 CONTINUE
      PT = GRDPOW(1,IANT)
      S4(JT) = SQRT((AMPAVE(JT,4)-AMPAVE(JT,2)**2)/AMPAVE(JT,2)**2)
      AMPAVE(JT,1) = AMPAVE(JT,1)/(0.5*SQRT(PI))/SQRT(PT)
      AMPAVE(JT,2) = AMPAVE(JT,2)/PT
      AMPAVE(JT,3) = AMPAVE(JT,3)/(0.75*SQRT(PI))/SQRT(PT)**3
      AMPAVE(JT,4) = AMPAVE(JT,4)/2.0/PT**2
      AMP = 0.5*ALOG(PT)
      CHIBAR(JT,2) = (CHIBAR(JT,2)-2.0*CHIBAR(JT,1)*AMP+AMP**2)/
     1 X2NORM
      CHIBAR(JT,1) = (CHIBAR(JT,1)-AMP)/X1NORM
```

Table 19. MEASR1 Fortran code (Continued).

```
C
C******  MEASURED FREQUENCY SELECTIVE BANDWIDTH
C
      AVETAU = AVETAU/AVENRG
      SIGTAU = SIGTAU/AVENRG - AVETAU**2
      FOMEAS = 1.0E30
      IF(SIGTAU .GT. 0.0)FOMEAS = 1.0/(TWOPI*SQRT(SIGTAU))
      AVENRG = AVENRG/NTIMES
      PO(IANT) = PO(IANT)/NTIMES
C
C******  WRITE MEASURED SIGNAL PARAMETERS VERSUS DELAY
C
      WRITE(IWRITE,2100)
 2100 FORMAT(/2X,'MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY',/4X,
     1  'NORMALIZED TO ENSEMBLE VALUES',/4X,
     2  'POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN',/4X,
     3  'J = T: STATISTICS OF COMPOSITE SIGNAL',//4X,
     4  'J',5X,'POW(J)',5X,'<A>',3X,'<A^2>',3X,'<A^3>',3X,
     5  '<A^4>',6X,'S4',3X,'<CHI>',1X,'<CHI^2>')
      DO 80 J=1,NDELAY
          JJ = J - 1
          WRITE(IWRITE,2110)JJ,G(J,IANT),(AMPAVE(J,M),M=1,4),S4(J),
     1      CHIBAR(J,1),CHIBAR(J,2)
 2110     FORMAT(2X,I3,1X,1PE10.3,7(1X,0PF7.4))
   80 CONTINUE
      WRITE(IWRITE,2111)GRDPOW(2,IANT),(AMPAVE(JT,M),M=1,4),S4(JT),
     1  CHIBAR(JT,1),CHIBAR(JT,2)
 2111 FORMAT(4X,'T',1X,1PE10.3,7(1X,0PF7.4))
C
C******  MEASURED REALIZATION PARAMETERS
C
      TOTLOS = -10.0*ALOG10(AVENRG)
      WRITE(IWRITE,2120)PA(IANT),AVENRG,SLOSDB(IANT),TOTLOS,FA(IANT),FOMEAS
 2120 FORMAT(/2X,'REALIZATION SIGNAL PARAMETERS:',12X,
     1  'ENSEMBLE',6X,'MEASURED'//4X,
     2  'MEAN POWER OF REALIZATION           = ',F10.6,4X,F10.6/4X,
     3  'TOTAL SCATTERING LOSS (DB)          = ',F10.3,4X,F10.3/4X,
     4  'FREQUENCY SELECTIVE BANDWIDTH (HZ)  = ',1PE10.3,4X,E10.3)
      WRITE(IWRITE,2121)TAUA(IANT),TAU0,N0,XTAU0
 2121 FORMAT(4X,
     1  'DECORRELATION TIME (SEC)            = ',1PE10.3,4X,E10.3/4X,
     2  'NUMBER OF SAMPLES PER DECORR. TIME  = ',I10,4X,0PF10.3)
      GRDLOS = 10.0*ALOG10(GRDPOW(3,IANT)/GRDPOW(1,IANT))
      WRITE(IWRITE,2130)GRDPOW(1,IANT),GRDLOS,GRDPOW(2,IANT)
 2130 FORMAT(/2X,'MEAN POWER IN GRID',//4X,
     1  'POWER IN KX-KY GIRD      = ',F10.6/4X,
     2  'POWER LOSS OF GRID (DB)  = ',F10.3/4X,
     3  'POWER IN DELAY GRID      = ',F10.6)
  100 CONTINUE
      IF(NUMANT .GT. 1)THEN
C
C******  ANTENNA OUTPUT VOLTAGE CROSS CORRELATIONS - ENSEMBLE VALUES
C
```

Table 19. MEASR1 Fortran code (Continued).

```
         WRITE(IWRITE,2200)
 2200    FORMAT(1H1,1X,'ENSEMBLE ANTENNA OUTPUT CROSS CORRELATION')
         WRITE(IWRITE,2210) (RHOAMP(II),II=1,NUMANT)
 2210    FORMAT(/4X,'AMPLITUDE OF CROSS CORRELATION',//
     1     5X,'N',2X,8A10)
         DO 120 IANT=1,NUMANT
             DO 110 JANT=1,NUMANT
                 TEMP(JANT) = CABS(CROSS(IANT,JANT))
  110        CONTINUE
             WRITE(IWRITE,2220)IANT,(TEMP(JANT),JANT=1,NUMANT)
 2220        FORMAT(4X,I2,2X,8F10.6)
  120    CONTINUE
         WRITE(IWRITE,2230) (RHOPHS(II),II=1,NUMANT)
 2230    FORMAT(/4X,'PHASE (RADIANS) OF CROSS CORRELATION',//
     1     5X,'N',2X,8A10)
         DO 140 IANT=1,NUMANT
             DO 130 JANT=1,NUMANT
               TEMP(JANT) =
     1         ARCTAN(AIMAG(CROSS(IANT,JANT)),REAL(CROSS(IANT,JANT)))
  130        CONTINUE
             WRITE(IWRITE,2220)IANT,(TEMP(JANT),JANT=1,NUMANT)
  140    CONTINUE
C
C       ANTENNA OUTPUT VOLTAGE CROSS CORRELATIONS - MEASURED VALUES
C
         DO 170 JANT=1,NUMANT
             DO 160 KANT=JANT,NUMANT
                 CROSS(JANT,KANT) = (0.0,0.0)
                 DO 150 KK=1,NTIMES
                     CROSS(JANT,KANT) = CROSS(JANT,KANT) +
     1                 HA(KK,JANT)*CONJG(HA(KK,KANT))
  150            CONTINUE
                 RHONOR = NTIMES*SQRT(P0(JANT)*P0(KANT))
                 CROSS(JANT,KANT) = CROSS(JANT,KANT)/RHONOR
                 IF(KANT.GT.JANT)CROSS(KANT,JANT) = CROSS(JANT,KANT)
  160        CONTINUE
  170    CONTINUE
         WRITE(IWRITE,2240)
 2240    FORMAT(//2X,'MEASURED ANTENNA OUTPUT CROSS CORRELATION')
         WRITE(IWRITE,2210) (RHOAMP(II),II=1,NUMANT)
         DO 190 IANT=1,NUMANT
             DO 180 JANT=1,NUMANT
                 TEMP(JANT) = CABS(CROSS(IANT,JANT))
  180        CONTINUE
             WRITE(IWRITE,2220)IANT,(TEMP(JANT),JANT=1,NUMANT)
  190    CONTINUE
         WRITE(IWRITE,2230) (RHOPHS(II),II=1,NUMANT)
         DO 210 IANT=1,NUMANT
             DO 200 JANT=1,NUMANT
               TEMP(JANT) =
     1         ARCTAN(AIMAG(CROSS(IANT,JANT)),REAL(CROSS(IANT,JANT)))
  200        CONTINUE
             WRITE(IWRITE,2220)IANT,(TEMP(JANT),JANT=1,NUMANT)
  210    CONTINUE
     END IF
     RETURN
     END
```

Table 20. READER Fortran code – ACIRF version.

```
      SUBROUTINE READER(RESET,NDELAY,NTIMES,NFILE,IUNIT,H)
C
C********************* VERSION 1.5 *** 08 JAN 1990 ****************************
C*********************        ACIRF VERSION         ****************************
C
C  THIS SUBROUTINE READS A COMPLEX ARRAY FROM A FILE ON UNIT=IUNIT
C  ONE DELAY ARRAY IS READ FOR EACH CALL WITH RESET = .FALSE.
C  THE INITIAL HEADER RECORDS ARE READ WITH RESET = .TRUE.
C
C  INPUTS FROM ARGUMENT LIST:
C
C  RESET  = LOGICAL FLAG
C  NDELAY = NUMBER OF DELAY SAMPLES IN IMPULSE RESPONSE FUNCTION
C         = DIMENSION OF H ARRAY
C  NTIMES = NUMBER OF TIME  SAMPLES IN IMPULSE RESPONSE FUNCTION
C  NFILE  = FILE NUMBER (ONE PER ANTENNA)
C  IUNIT  = UNIT OF FILE
C
C  OUTPUTS TO ARGUMENT LIST:
C
C  H      = IMPULSE RESPONSE FUNCTION ARRAY (DIMENSION NDELAY)
C
C  COMMON BLOCK INPUTS:
C
C  RDATA1 = HEADER RECORD
C
C  PARAMETERS:
C
C  MAXBUF = MAXIMUM BUFFER SIZE (RECORD SIZE IN FILE = MAXBUF + 1)
C  MTENNA = MAXIMUM NUMBER OF ANTENNAS (EQUAL TO THE MAX NUMBER OF FILES)
C  NDATA1 = SIZE OF HEADER RECORD 1
C  NDATA2 = SIZE OF HEADER RECORD 2
C
C******************************************************************************
C
      SAVE
      INCLUDE 'ACIRF.SIZ'
      COMMON/HEADER/RDATA1(NDATA1),RDATA2(MTENNA,NDATA2)
      COMPLEX H(NDELAY),BUFFER(MAXBUF/2,MTENNA)
      DIMENSION HEADER(NDATA1),NUMBER(MTENNA),DUMMY(NDATA2)
      LOGICAL RESET
      CHARACTER*80 IDENT
C
C******  IF RESET = .TRUE. READ HEADER RECORDS AT BEGINNING OF FILE
C
      IF(RESET)THEN
          REWIND IUNIT
          READ(IUNIT) MDENT,IDENT(1:MIN0(MDENT,80))
          READ(IUNIT) MDATA1,(HEADER(II),II=1,MIN0(MDATA1,NDATA1))
          READ(IUNIT) MDATA2,(DUMMY (II),II=1,MIN0(MDATA2,NDATA2))
          MDATA1 = MIN0(MDATA1,NDATA1)
          DO 10 NN=1,MDATA1
```

Table 20. READER Fortran code - ACIRF version (Continued).

```
C
C  FATAL ERROR IF HEADER RECORD DOES NOT AGREE WITH RDATA1
C
              IF(HEADER(NN) .NE. RDATA1(NN))THEN
                  WRITE(*,4000)NN,HEADER(NN),NN,RDATA1(NN)
 4000             FORMAT(2X,'FATAL ERROR REREADING HEADER RECORD',/2X,
     1             'WORD ',I2,' OF HEADER RECORD IS        ',1PE12.5/2X,
     2             'WORD ',I2,' OF HEADER RECORD SHOULD BE ',E12.5)
                  STOP ' READER EXECUTION TERMINATED'
              END IF
   10     CONTINUE
          NTBUF = (MAXBUF/2)/NDELAY
          MAXCX = NTBUF*NDELAY
          DO 20 NN=1,MTENNA
              NUMBER(NN) = 0
   20     CONTINUE
          RESET = .FALSE.
          RETURN
      END IF
      NUMBER(NFILE) = NUMBER(NFILE) + 1
C
C  COUNT NUMBER OF CALLS TO PROGRAM
C  FATAL ERROR IF ATTEMPT TO READ BEYOND END-OF-FILE
C
      IF(NUMBER(NFILE) .GT. NTIMES)THEN
          WRITE(*,4001)NUMBER(NFILE),NTIMES
 4001     FORMAT(2X,'ATTEMPT TO READ BEYOND END-OF-FILE' /2X,
     1       'TIME INDEX = ',I10/2X,'MAX  INDEX = ',I10)
          STOP ' READER EXECUTION TERMINATED'
      END IF
C
C  READ DATA INTO A BUFFER
C
      IF(MOD(NUMBER(NFILE),NTBUF) .EQ. 1)THEN
          READ(IUNIT) MDATA1,(HEADER(NN),NN=1,MINO(MDATA1,NDATA1))
          READ(IUNIT) NRLWRD,
     1       (BUFFER(KK,NFILE),KK=1,MINO(NRLWRD/2,MAXBUF/2))
          MDATA1 = MINO(MDATA1,NDATA1)
          DO 30 NN=1,MDATA1
C
C  FATAL ERROR IF HEADER RECORD DOES NOT AGREE WITH RDATA1
C
              IF(HEADER(NN) .NE. RDATA1(NN))THEN
                  WRITE(*,4000)NN,HEADER(NN),NN,RDATA1(NN)
                  STOP ' READER EXECUTION TERMINATED'
              END IF
   30     CONTINUE
      END IF
C
C  READ BUFFERED DATA INTO ARRAY
C
      JOFFST = MOD((NUMBER(NFILE)-1)*NDELAY,MAXCX)
      DO 40 JJ=1,NDELAY
          H(JJ) = BUFFER(JOFFST+JJ,NFILE)
   40 CONTINUE
      RETURN
      END
```

Table 21. TAUGD1 Fortran code.

```
      SUBROUTINE TAUGD1(NDRQD)
C
C********************* VERSION 1.5 *** 08 JAN 1990 ****************************
C
C  THIS SUBROUTINE COMPUTES THE ENERGY IN DELAY GRID CELLS AND:
C    STOPS ACIRF IF THERE ARE TOO FEW DELAY GRID CELLS
C    PRINTS A WARNING IF THERE ARE TOO MANY DELAY GRID CELLS
C
C**********************  SET ARRAY SIZES  *************************************
C
      INCLUDE 'ACIRF.SIZ'
C
C******************************************************************************
C
      EXTERNAL PIRF1
      COMMON/ANTENA/NUMANT,XPOS(MTENNA),YPOS(MTENNA)
      COMMON/ANTNUM/IANT
      COMMON/CHANEL/DLX,DLY,F0,CXT,CYT
      COMMON/GRDSIZ/DELTAU,DELTKX,DELTKY,TAU0DF
      COMMON/INTGRL/NTASUB
      COMMON/REALZN/NKX,NKY,NDELAY,NTIMES,N0,NFMAX
      COMMON/POWERS/G(MDELAY,MTENNA),GRDPOW(3,MTENNA)
      PARAMETER (PI=3.141592654)
      PARAMETER (SMALL=1.0E-10,BIG=5.0,RQD=0.975)
      LOGICAL STOPER
C
C******************************************************************************
C
C******  COMPUTE ENERGY IN ANGULAR DELAY GRID
C
      OMEGAC = 2.0*PI*F0
      DO 20 IANT=1,NUMANT
          GRDPOW(2,IANT) = 0.0
   20 CONTINUE
      DO 40 J=1,NDELAY
          TAU1 = OMEGAC*(J-1)*DELTAU
          TAU2 = TAU1 + OMEGAC*DELTAU
          DO 30 IANT=1,NUMANT
              IF(TAU1 .LE. SMALL .AND. TAU2 .GE. BIG)THEN
                  G(J,IANT) = GRDPOW(3,IANT)
                  GRDPOW(2,IANT) = GRDPOW(2,IANT) + G(J,IANT)
                  GO TO 30
              END IF
              G(J,IANT) = SIMPSN(PIRF1,TAU1,TAU2,NTASUB)
              GRDPOW(2,IANT) = GRDPOW(2,IANT) + G(J,IANT)
   30     CONTINUE
   40 CONTINUE
C
C******  TEST DELAY GRID FOR TOO LITTLE ENERGY OR EXCESS DELAYS
C
      STOPER = .FALSE.
      TAUMAX = 0.0
      NDRQD  = 0
      DO 80 IANT=1,NUMANT
          EREQ = RQD*GRDPOW(3,IANT)
          IF(GRDPOW(2,IANT) .LT. EREQ)THEN
```

Table 21. TAUGD1 Fortran code (Continued).

```
C
C  NOT ENOUGH ENERGY IN GRID, CALCULATE REQUIRED MAXIMUM DELAY
C
               STOPER = .TRUE.
               TAU1 = 0.0
               TAU2 = OMEGAC*NDELAY*DELTAU
               MAXITR = 1 + ALOG(BIG/TAU2)/ALOG(1.1)
               ETAU1 = SIMPSN(PIRF1,TAU1,TAU2,NDELAY*NTASUB)
               TAU1 = TAU2
               DO 60 ITER=1,MAXITR
                   TAU2 = 1.1*TAU1
                   ETAU2 = ETAU1 + SIMPSN(PIRF1,TAU1,TAU2,NTASUB)
                   IF(ETAU2 .GE. EREQ)GO TO 65
                   TAU1  = TAU2
                   ETAU1 = ETAU2
   60          CONTINUE
   65          TAU2 = TAU1 + (EREQ-ETAU1)*(TAU2-TAU1)/(ETAU2-ETAU1)
               TAUMAX = AMAX1(TAUMAX,TAU2)
           ELSE
C
C  ENOUGH ENERGY, SEE IF THERE ARE TOO MANY DELAY BINS
C
               ETAU2 = 0.0
               DO 70 J=1,NDELAY
                   ETAU2 = ETAU2 + G(J,IANT)
                   IF(ETAU2 .GE. EREQ) GO TO 75
   70          CONTINUE
   75          NDRQD = MAX0(NDRQD,J)
           END IF
   80 CONTINUE
C
C****** NOT ENOUGH ENERGY IN DELAY GRID - STOP ACIRF
C
      IF(STOPER)THEN
          TAUINP  = NDELAY*DELTAU
          TAURQD  = TAUMAX/OMEGAC
          NDRQD = 1.0 + TAURQD/DELTAU
          WRITE(*,4011)
          WRITE(*,4012)TAUINP,TAURQD,NDELAY,NDRQD
 4011     FORMAT(2X,'******   DELAY GRID ERROR   ******')
 4012     FORMAT(4X,'INSUFFICIENT DELAY IN DELAY GRID'/4X,
     1      'INPUT    DELAY  = ',1PE12.4,' = NDELAY*DELTAU'/4X,
     2      'REQUIRED DELAY  = ',E12.4/4X,
     3      'INPUT    NDELAY = ',I12/4X,
     4      'REQUIRED NDELAY = ',I12,' FOR FIXED DELTAU')
          STOP '  ACIRF EXECUTION TERMINATED'
      END IF
C
C****** TOO MANY DELAY BINS - PRINT WARNING
C
      IF(NDRQD .LT. NDELAY)WRITE(*,4015)NDRQD
 4015 FORMAT(/2X,'**********   WARNING   **********'/2X,
     1  'ONLY ',I4,' DELAY BINS ARE REQUIRED'/2X,
     2  '***********************************')
      RETURN
      END
```

Table 22. WRITER Fortran code - ACIRF version.

```
      SUBROUTINE WRITER(RESET,NDELAY,NTIMES,NFILE,IUNIT,H)
C
C********************* VERSION 1.3 *** 08 JAN 1990 ****************************
C*********************         ACIRF VERSION         ****************************
C
C  THIS SUBROUTINE WRITES A COMPLEX ARRAY INTO A FILE ON UNIT=IUNIT
C
C
C  INPUTS:
C
C  RESET  = LOGICAL FLAG - INITIALIZATION IF RESET = .TRUE.
C  H      = COMPLEX*8 ARRAY
C  NDELAY = NUMBER OF DELAY SAMPLES IN IMPULSE RESPONSE FUNCTION
C  NTIMES = NUMBER OF TIME  SAMPLES IN IMPULSE RESPONSE FUNCTION
C  NFILE  = FILE NUMBER
C  IUNIT  = UNIT NUMBER OF FILE
C
C  COMMON BLOCK INPUTS:
C
C  RDATA1 = HEADER RECORD
C
C  PARAMETERS:
C
C  MAXBUF = MAXIMUM BUFFER SIZE (RECORD SIZE IN FILE = MAXBUF + 1)
C  MTENNA = MAXIMUM NUMBER OF ANTENNAS (EQUAL TO MAXIMUM NUMBER OF FILES)
C  NDATA1 = SIZE OF HEADER RECORD 1
C  NDATA2 = SIZE OF HEADER RECORD 2
C
C******************************************************************************
C
      SAVE
      INCLUDE 'ACIRF.SIZ'
      LOGICAL RESET
      COMMON/HEADER/RDATA1(NDATA1),RDATA2(MTENNA,NDATA2)
      COMPLEX H(NDELAY),BUFFER(MAXBUF/2,MTENNA)
      DIMENSION NUMBER(MTENNA)
C
C******  IF RESET .EQ. TRUE THEN INITIALIZE
C
      IF(RESET)THEN
          NTBUF  = (MAXBUF/2)/NDELAY
          NCXWRD = NTBUF*NDELAY
          NRLWRD = 2*NCXWRD
          DO 10 NN=1,MTENNA
              NUMBER(NN) = 0
   10     CONTINUE
          RESET = .FALSE.
          RETURN
      END IF
C
C  COUNT NUMBER OF CALLS - ONE PER TIME SAMPLE
C
```

Table 22. WRITER Fortran code - ACIRF version (Continued).

```
      NUMBER(NFILE) = NUMBER(NFILE) + 1
      JOFFST = MOD((NUMBER(NFILE)-1)*NDELAY,NCXWRD)
C
C  WRITE DELAY SAMPLES INTO BUFFER
C
      DO 20 JJ=1,NDELAY
          BUFFER(JOFFST+JJ,NFILE) = H(JJ)
   20 CONTINUE
      IRITER = MOD(NUMBER(NFILE),NTBUF)
C
C  WRITE BUFFER SAMPLES INTO FILE WHEN BUFFER IS FULL
C
      IF(IRITER .EQ. 0)THEN
          WRITE(IUNIT) NDATA1,(RDATA1(NN),NN=1,NDATA1)
          WRITE(IUNIT) NRLWRD,(BUFFER(KK,NFILE),KK=1,NCXWRD)
      END IF
C
C  WRITE BUFFER SAMPLES INTO FILE WHEN AT END OF REALIZATION
C
      IF(NUMBER(NFILE) .EQ. NTIMES .AND. IRITER .NE. 0)THEN
          LFTCX = MOD(NTIMES,NTBUF)*NDELAY
          LFTRL = 2*LFTCX
          WRITE(IUNIT) NDATA1,(RDATA1(NN),NN=1,NDATA1)
          WRITE(IUNIT) LFTRL,(BUFFER(KK,NFILE),KK=1,LFTCX)
      END IF
      RETURN
      END
```

Table 23. ARCTAN Fortran code.

```
      FUNCTION ARCTAN(X,Y)
C
C********************* VERSION 1.2 *** 08 JAN 1988 ****************************
C
C  ARC-TANGENT FUNCTION
C
C******************************************************************************
C
      PARAMETER (SMALL=1.0E-20)
      IF(ABS(X) .LT. SMALL .AND. ABS(Y) .LT. SMALL)THEN
          ARCTAN = 0.0
      ELSE
          ARCTAN = ATAN2(X,Y)
      END IF
      RETURN
      END
```

Table 24. BI0EXP Fortran code.

```
      FUNCTION BI0EXP(X)
C
C********************* VERSION 1.0 *** 25 NOV 1987 ****************************
C
C  EXP(-X)*I0(X) BESSEL FUNCTION (ABRAMOWITZ AND STEGUN 9.8.1 AND 9.8.2)
C
C******************************************************************************
C
      DIMENSION C1(6),C2(9)
      DATA C1/3.5156229,3.0899424,1.2067492,0.2659732,0.0360768,
     1        0.0045813/
      DATA C2/0.39894228,0.01328592,0.00225319,-0.00157565,0.00916281,
     1        -0.02057706,0.02635537,-0.01647633,0.00392377/
      T = X/3.75
      BI0EXP = 0.0
      IF(T .LE. 1.0)THEN
          BI0 = 1.0
          DO 10 I=1,6
              BI0 = BI0 + C1(I)*T**(2*I)
   10     CONTINUE
          BI0EXP = EXP(-X)*BI0
      ELSE IF(T .GT. 1.0)THEN
          BI0 = 0.0
          DO 20 I=1,9
              BI0 = BI0 + C2(I)/T**(I-1)
   20     CONTINUE
          BI0EXP = BI0/SQRT(X)
      END IF
      RETURN
      END
```

Table 25. PIRF1 Fortran code.

```
      FUNCTION PIRF1(TAU)
C
C******************** VERSION 2.4 *** 25 APR 1991 ****************************
C
C  THIS FUNCTION CALCULATES THE POWER IMPULSE RESPONSE FUNCTION
C  OF THE SIGNAL AT THE ANTENNA OUTPUT.  THE CALCULATION INCLUDES
C  THE EFFECTS OF NONZERO POINTING ANGLES.
C  THE BESSEL FUNCTION SERIES IS CALCULATED BY BACKWARD RECURSION.
C
      INCLUDE 'ACIRF.SIZ'
C
C******************************************************************************
C
      COMMON/ANTNUM/IANT
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/POWIRF/CPIRF(5,MTENNA),G0(MTENNA)
      PARAMETER (SMALL=1.0E-10)
      LOGICAL LIMIT
      DATA LIMIT /.FALSE./
      SAVE LIMIT
      X = CPIRF(2,IANT)*TAU/2.0
      Y = CPIRF(3,IANT)*SQRT(TAU)/2.0
      W = (CPIRF(1,IANT) - CPIRF(2,IANT))*TAU -
     1  CPIRF(3,IANT)*SQRT(TAU)
      IF(X .LE. SMALL .OR. Y .LE. SMALL) THEN
          PIRF1 = EXP(-W)*BI0EXP(2.0*X)*BI0EXP(2.0*Y)
     1          *G0(IANT)*CPIRF(5,IANT)
          RETURN
      END IF
      Z = CPIRF(4,IANT)
      KM = 10.0*MAX(X,Y/2.0)
      KM = MAX(KM,10)
      IF(KM .GT. 100)THEN
          KM = 100
          IF(.NOT.LIMIT)THEN
              LIMIT = .TRUE.
              WRITE(IWRITE,2000)
 2000         FORMAT(/2X,
     1          '*******************************************'/2X,
     2          'PROBLEM IN PIRF1 - POW(J) MAY BE UNRELIABLE'/2X,
     3          '*******************************************'/)
          END IF
      END IF
      RX = 0.0
      RY1 = 0.0
      RY2 = 0.0
      PIRF1 = 0.0
      DO 10 I=KM,1,-1
          RX = 1.0/(RX+FLOAT(I)/X)
          RY1 = 1.0/(RY2+FLOAT(2*I)/Y)
          RY2 = 1.0/(RY1+FLOAT(2*I-1)/Y)
          PIRF1 = (PIRF1 + COS(FLOAT(I)*Z))*RX*RY1*RY2
10    CONTINUE
      PIRF1 = EXP(-W)*BI0EXP(2.0*X)*BI0EXP(2.0*Y)*(1.0 + 2.0*PIRF1)
     1         *G0(IANT)*CPIRF(5,IANT)
      RETURN
      END
```

Table 26. CIRF Fortran code.

```
      PROGRAM CIRF
      PARAMETER (VERSON=1.11)
C
C******************** VERSION 1.11 *** 25 APR 1991 ***************************
C
C     THIS PROGRAM GENERATES REALIZATIONS OF THE CHANNEL IMPULSE RESPONSE.
C     NO ANTENNA EFFECTS ARE INCLUDED.  FREQUENCY SELECTIVE FADING, FLAT
C     FADING, AND NON-FADING CHANNELS MODELS MAY BE SELECTED.  FOR
C     FREQUENCY SELECTIVE FADING CHANNELS, EITHER THE FROZEN-IN OR TURBULENT
C     MODELS MAY BE SPECIFIED.
C
C     THIS ROUTINE WAS DESIGNED AND WRITTEN AND IS MAINTAINED BY:
C                    ROGER A. DANA
C                    MISSION RESEARCH CORPORATION
C                    735 STATE STREET
C                    P.O. DRAWER 719
C                    SANTA BARBARA, CALIFORNIA 93102
C                    (805)963-8761 EXT. 212
C
C     TESTING, BITS OF CODE, AND BLESSINGS BY
C                    LEON A. WITTWER
C                    DEFENSE NUCLEAR AGENCY
C                    ATMOSPHERIC EFFECTS DIVISION
C                    WASHINGTON, DC 20305
C                    (202)325-7028
C
C*********************  SET ARRAY SIZES  *************************************
C
C  MDELAY = MAXIMUM NUMBER OF DELAY SAMPLES
C  MTIMES = MAXIMUM NUMBER OF TIME SAMPLES
C  MAXBUF = MAXIMUM NUMBER OF WORDS IN A BUFFER
C           MAXIMUM RECORD SIZE = MAXBUF + 1
C
      INCLUDE 'CIRF.SIZ'
C
C**************************  INPUT DATA  *************************************
C
C  CHANNEL PARAMETERS
C
C    IFADE   = CHANNEL MODEL FLAG
C            = 0 CONSTANT IMPULSE RESPONSE FUNCTION (AWGN)
C            = 1 NOT ALLOWED IN CIRF (GENERAL MODEL WITH ANTENNAS)
C            = 2 FROZEN-IN MODEL
C            = 3 TURBULENT MODEL
C            = 4 FLAT FADING
C    F0      = FREQUENCY SELECTIVE BANDWIDTH (HZ)
C    TAU0    = DECORRELATION TIME (SECONDS)
C
C  REALIZATION PARAMETERS
C
C    KASE    = KASE NUMBER OF REALIZATION
C    IDENT   = ALPHANUMERIC IDENTIFIER FOR REALIZATION
C    DELTAU  = DELAY SAMPLE SIZE (SEC)
C    NDELAY  = NUMBER OF DELAY SAMPLES
C    NTIMES  = NUMBER OF TIME SAMPLES
C    IJSEED  = INITIAL RANDOM NUMBER SEED
```

Table 26. CIRF Fortran code (Continued).

```
C    NO       = NUMBER OF SAMPLES PER DECORRELATION TIME
C
C    NOTES:  GENERAL MODEL WITH ANTENNAS (IFADE=1) REQUIRES ACIRF CODE
C            CASE NUMBER FOR IDENTIFICATION ONLY
C            IDENT MUST BE <= 80 CHARACTERS
C            NTIMES MUST BE <= MTIMES
C            NDELAY MUST BE <= MDELAY
C            NDELAY= 1 FOR FLAT FADE OR AWGN MODELS
C            NTIMES DEFAULT = 1024
C            IJSEED DEFAULT = 9771975
C            NO DEFAULT = 10 (MUST BE >= 10)
C
C*******************************************************************************
C
      CHARACTER IDENT*80
      LOGICAL STOPER
      COMMON/CHANEL/TAU0,F0
      COMMON/GRDSIZ/DELTAU,TAUMIN,DELTAT,DELTAF
      COMMON/HEADER/RDATA1(NDATA1),RDATA2(NDATA2)
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/INTGRL/NTASUB
      COMMON/POWERS/G(MDELAY),GRDPOW(2)
      COMMON/RANSED/ISEED,JSEED
      COMMON/REALZN/NDELAY,NTIMES,ALPHA,N0
      PARAMETER (PI=3.141592654,TWOPI=2.0*PI)
      DATA STOPER/.TRUE./
C
C****** SET CONSTANTS AND FIXED PARAMETERS
C
      ALPHA  = 4.0
      NTASUB = 100
      INPUT  = 1
      IREAD  = 2
      IWRITE = 3
      IOFILE = 11
C
C*******************************************************************************
C
      OPEN(UNIT=IREAD ,FILE='CIRF.DAT',STATUS='OLD')
      OPEN(UNIT=IWRITE,FILE='CIRF.OUT',STATUS='NEW')
      CALL PREPRC(IREAD,INPUT)
      CLOSE(UNIT=IREAD)
C
C  CASE NUMBER AND IDENTIFICATION
C
      READ(INPUT,*)KASE,STOPER
      IDENT = ' '
      READ(INPUT,*)IDENT
      MDENT = LENGTH(IDENT)
C
C  CHANNEL PARAMETERS
C
      READ(INPUT,*)IFADE,F0,TAU0
      IF(IFADE .LT. 0 .OR. IFADE .EQ. 1 .OR. IFADE .GT. 4)THEN
          WRITE(*,4000)
 4000     FORMAT(2X,'****** INPUT ERROR ******')
          WRITE(*,4010)IFADE
 4010     FORMAT(2X,'IFADE MUST BE = 0,2,3,4',/2X,'IFADE = ',I10)
```

Table 26. CIRF Fortran code (Continued).

```
         STOP '  CIRF EXECUTION TERMINATED'
      END IF
C
C  REALIZATION PARAMETERS
C
      NTIMES = 1024
      IJSEED = 9771975
      READ(INPUT,*)NDELAY,DELTAU,NTIMES,IJSEED,NO
      CLOSE(UNIT=INPUT,STATUS='DELETE')
      JSEED = IAND(IJSEED,65535)
      ISEED = IAND(ISHFT(IJSEED,-16),65535)
      IF(IFADE .EQ. 4)THEN
          FO    = 1.0E30
          NDELAY = 1
      END IF
      IF(IFADE .EQ. 0)THEN
          NDELAY = 1
          FO     = 1.0E30
          TAUO   = 1.0E30
      END IF
C
C  NTIMES MUST BE >= 1 AND <= MTIMES
C
      IF(NTIMES .LT. 1 .OR. NTIMES .GT. MTIMES)THEN
          WRITE(*,4000)
          WRITE(*,4050)NTIMES,MTIMES
 4050     FORMAT(2X,'NTIMES MUST BE >= 1 AND <= MTIMES',/2X,
     1      'NTIMES = ',I10/2X,'MTIMES = ',I10)
          STOP '  CIRF EXECUTION TERMINATED'
      END IF
C
C  NDELAY MUST BE >= 1 AND <= MDELAY
C
      IF(NDELAY .LT. 1 .OR. NDELAY .GT. MDELAY)THEN
          WRITE(*,4000)
          WRITE(*,4060)NDELAY,MDELAY
 4060     FORMAT(2X,'NDELAY MUST BE >= 1 AND <= MDELAY',/2X,
     1      'NDELAY = ',I10/2X,'MDELAY = ',I10)
          STOP '  CIRF EXECUTION TERMINATED'
      END IF
C
C  THERE MUST BE AT LEAST 10 SAMPLES PER DECORRELATION DISTANCE OR TIME
C
      IF(NO .LT. 10)THEN
          WRITE(*,4000)
          WRITE(*,4070)NO
 4070     FORMAT(2X,'NO MUST BE >= 10',/2X,'NO = ',I10)
          IF(STOPER)STOP '  CIRF EXECUTION TERMINATED'
      END IF
C
C  THERE MUST BE AT LEAST 100 DECORRELATION TIMES IN EACH REALIZATION
C
      NTMIN = 100*NO
      IF(NTIMES .LT. NTMIN)THEN
          WRITE(*,4000)
          WRITE(*,4080)NTIMES,NTMIN,MTIMES
```

Table 26. CIRF Fortran code (Continued).

```
 4080     FORMAT(2X,'THERE MUST BE AT LEAST 100 DECORRELATION TIMES ',
    1       'IN EACH REALIZATION',/2X,'NTIMES = ',I10/2X,
    2       'REQUIRED NTIMES = ',I10/2X,'MAXIMUM NTIMES = ',I10)
          IF(STOPER)STOP    CIRF EXECUTION TERMINATED'
      END IF
C
C******  CALCULATE GRID SIZES
C
      DELTAT = TAU0/N0
      DELTAF = TWOPI/(NTIMES*DELTAT)
      TAUMIN = DELTAU/2.0
      IF(IFADE .EQ. 2)TAUMIN = -0.25/(TWOPI*F0)
      IF(IFADE .EQ. 0 .OR. IFADE .EQ. 4)THEN
          GRDPOW(1) = 1.0
          GRDPOW(2) = 1.0
          G(1)      = 1.0
          NDRQD     = 1
      ELSE
          CALL TAUGD2(IFADE,NDRQD)
      END IF
C
C******  PRINT INPUT DATA
C
      WRITE(IWRITE,2000)VERSON,KASE
 2000 FORMAT(2X,'CIRF CHANNEL SIMULATION VERSION ',F6.2/4X,
    1  'CASE NUMBER ',I10)
      IF(IFADE .EQ. 0)WRITE(IWRITE,2001)
      IF(IFADE .EQ. 2)WRITE(IWRITE,2002)
      IF(IFADE .EQ. 3)WRITE(IWRITE,2003)
      IF(IFADE .EQ. 4)WRITE(IWRITE,2004)
 2001 FORMAT(4X,'CONSTANT IMPULSE RESPONSE FUNCTION')
 2002 FORMAT(4X,'TEMPORAL VARIATION FROM FROZEN-IN MODEL')
 2003 FORMAT(4X,'TEMPORAL VARIATION FROM TURBULENT MODEL')
 2004 FORMAT(4X,'FLAT FADING WITH 1/F**4 DOPPLER SPECTRUM')
      WRITE(IWRITE,2009) IDENT(1:MDENT)
 2009 FORMAT(4X,'REALIZATION IDENTIFICATION:',/4X,A)
      WRITE(IWRITE,2010)F0,TAU0
 2010 FORMAT(/2X,'CHANNEL PARAMETERS',/4X,
    1   'FREQUENCY SELECTIVE BANDWIDTH  (HZ) = ',1PE10.3/4X,
    2   'DECORRELATION TIME (SEC)            = ',E10.3)
      WRITE(IWRITE,2020)NDELAY,DELTAU,NTIMES,N0,IJSEED
 2020 FORMAT(/2X,'REALIZATION PARAMETERS',/4X,
    1   'NUMBER OF DELAY SAMPLES              = ',I10/4X,
    2   'DELAY SAMPLE SIZE (SEC)              = ',1PE10.3/4X,
    3   'NUMBER OF TEMPORAL SAMPLES           = ',I10/4X,
    4   'NUMBER OF TEMPORAL SAMPLES PER TAU0 = ',I10/4X,
    5   'INITIAL RANDOM NUMBER SEED        = ',I12)
      IF(NDRQD .LT. NDELAY)WRITE(IWRITE,2025)NDRQD
 2025 FORMAT(/2X,'**********   WARNING   **********'/2X,
    1   'ONLY ',I4,' DELAY BINS ARE REQUIRED'/2X,
    2   '**********************************')
C
C******  OPEN BINARY FILES FOR REALIZATIONS
C
      DO 100 II=1,NDATA1
          RDATA1(II) = 0.0
  100 CONTINUE
```

Table 26. CIRF Fortran code (Continued).

```
      DO 105 IJ=1,NDATA2
          RDATA2(IJ) = 0.0
  105 CONTINUE
      RDATA1( 1) = 2.0
      RDATA1( 2) = KASE
      RDATA1( 4) = TAU0
      RDATA1( 5) = F0
      RDATA1( 9) = 1.0
      RDATA1(13) = NTIMES*DELTAT
      RDATA1(14) = NTIMES
      RDATA1(15) = DELTAT
      RDATA1(16) = N0
      RDATA1(20) = NDELAY
      RDATA1(21) = TAUMIN
      RDATA1(22) = DELTAU
      RDATA1(23) = ISEED
      RDATA1(24) = JSEED
      RDATA1(25) = MAXBUF
      RDATA1(28) = VERSON
      OPEN(UNIT=IOFILE,FILE='CIRF.BIN',STATUS='NEW',FORM='UNFORMATTED')
      WRITE(IOFILE) MDENT,IDENT(1:MDENT)
      WRITE(IOFILE) NDATA1,(RDATA1(II),II=1,NDATA1)
      WRITE(IOFILE) NDATA2,(RDATA2(II),II=1,NDATA2)
C
C******  CHANNEL SIMULATORS
C
      IF(IFADE .EQ. 0)CALL CHANL0
      IF(IFADE .EQ. 2)CALL CHANL2
      IF(IFADE .EQ. 3)CALL CHANL3
      IF(IFADE .EQ. 4)CALL CHANL4
C
C******  MEASURE IMPULSE RESPONSE FUNCTION STATISTICS
C
      CALL MEASR2
      IJSEED = IOR(ISHFT(ISEED,16),JSEED)
      WRITE(IWRITE,2100)IJSEED
 2100 FORMAT(/2X,'FINAL RANDOM NUMBER SEED = ',I12)
      CLOSE(UNIT=IWRITE)
      STOP '  CIRF COMPLETED'
      END
```

Table 27. CHANL0 Fortran code.

```
      SUBROUTINE CHANLO
C
C********************* VERSION 1.2 *** 08 JAN 1990 ****************************
C
C  THIS SUBROUTINE GENERATES A CONSTANT IMPULSE RESPONSE FUNCTION
C
C******************************************************************************
C
      COMMON/GRDSIZ/DELTAU,TAUMIN,DELTAT,DELTAF
      COMMON/REALZN/NDELAY,NTIMES,ALPHA,NO
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMPLEX HA
      LOGICAL RESET
C
C****** RESET WRITER ROUTINE
C
      RESET = .TRUE.
      CALL WRITER(RESET,NDELAY,NTIMES,IOFILE,HA)
C
C******  CALCULATE CONSTANT IMPULSE
C
      HA = CMPLX(1.0/DELTAU,0.0)
C
C******  SET IMPULSE RESPONSE FUNCTION TO HA FOR ALL TIMES
C
      DO 20 NT=1,NTIMES
          CALL WRITER(RESET,NDELAY,NTIMES,IOFILE,HA)
          IF(MOD(NT,NTIMES/NO) .EQ. 0)THEN
              WRITE(*,3000)NT
 3000         FORMAT(2X,'TIME ',I5,' COMPLETE')
          END IF
   20 CONTINUE
      RETURN
      END
```

Table 28. CHANL2 Fortran code.

```
      SUBROUTINE CHANL2
C
C********************* VERSION 2.3 *** 08 JAN 1990 ******************************
C
C  THIS SUBROUTINE CALCULATES IMPULSE RESPONSE FUNCTION USING
C  FROZEN-IN APPROXIMATION WITHOUT ANTENNA EFFECTS.
C
C**********************  SET ARRAY SIZES  ***************************************
C
      INCLUDE 'CIRF.SIZ'
C
C*******************************************************************************
C
      EXTERNAL TAUPSD
      COMMON/CHANEL/TAU0,F0
      COMMON/GRDSIZ/DELTAU,TAUMIN,DELTAT,DELTAF
      COMMON/IMPULS/HA(MTIMES,MDELAY),HB(MTIMES),HC(MDELAY)
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/INTGRL/NTASUB
      COMMON/POWERS/G(MDELAY),GRDPOW(2)
      COMMON/PSDCOM/OMEGAT,S0
      COMMON/RANSFD/ISEED,JSEED
      COMMON/REALZN/NDELAY,NTIMES,ALPHA,N0
      COMPLEX AWGN,HA,HB,HC
      PARAMETER (PI=3.141592654,TWOPI=2.0*PI)
      PARAMETER (SMALL=1.0E-10)
C
C  PRECOMPUTE COEFFICIENTS OF GENERALIZED POWER SPECTRUM
C
      S0 = TAU0*SQRT(ALPHA/(PI*SQRT(2.0)))
      OMEGAC = TWOPI*F0*SQRT(1.0+1.0/ALPHA**2)
      MIDDOP = NTIMES/2 + 1
      GRDPOW(1) = 0.0
      IFFT = -1
      LOGNT  = (ALOG(FLOAT(NTIMES))/ALOG(2.0) + 0.5)
C
C  FIND DELAY BIN WITH LARGEST ENERGY
C
      GMAX  = 0.0
      JGMAX = 1
      DO 10 J=0,NDELAY-1
          IF(G(J+1) .GT. GMAX)THEN
              JGMAX = J+1
              GMAX  = G(J+1)
          END IF
   10 CONTINUE
C
C  START LOOP OVER DELAY
C
      DO 50 J=0,NDELAY-1
C
C  SET IMPULSE RESPONSE FUNCTION TO ZERO IF ENERGY IN DELAY BIN IS SMALL
C
          IF(G(J+1) .LT. SMALL*G(JGMAX))THEN
              DO 20 KK=1,NTIMES
                  HB(KK) = (0.0,0.0)
```

Table 28. CHANL2 Fortran code (Continued).

```
  20          CONTINUE
              GO TO 35
          END IF
C
C  CALCULATE ENERGY IN A KX-KY GRID CELL AND GENERATE RANDOM VOLTAGE
C  ZERO DC FREQUENCY COMPONENT
C
          TAU  = TAUMIN + J*DELTAU
          TAU1 = OMEGAC*(TAU - DELTAU/2.0)
          TAU2 = OMEGAC*(TAU + DELTAU/2.0)
          DO 30 KK=1,MIDDOP
              KN = NTIMES - KK + 2
              DOP = (KK-1)*DELTAF
              OMEGAT = (DOP*TAU0/2.0)**2
              ENRG = (DELTAF/TWOPI)*SIMPSN(TAUPSD,TAU1,TAU2,NTASUB)
              GRDPOW(1) = GRDPOW(1) + ENRG
              IF(KK .GT. 1)THEN
                  HB(KK) = SQRT(ENRG)*AWGN(ISEED,JSEED)/DELTAU
              ELSE
                  HB(KK) = (0.0,0.0)
              END IF
              IF(KK .NE. 1 .AND. KK .NE. MIDDOP)THEN
                  GRDPOW(1) = GRDPOW(1) + ENRG
                  HB(KN) = SQRT(ENRG)*AWGN(ISEED,JSEED)/DELTAU
              END IF
  30      CONTINUE
C
C  ZERO DC TERM
C
      HB(1) = (0.0,0.0)
C
C  FAST FOURIER TRANSFORM FROM DOPPLER TO TIME
C
          CALL FFT(HB,LOGNT,NTIMES,IFFT)
C
C  OUTPUT IMPULSE RESPONSE FUNCTION INTO BINARY FILE
C
  35      DO 40 KK=1,NTIMES,MAXBUF/2
              NK = MIN0(NTIMES,KK+MAXBUF/2-1)
              WRITE(IOFILE) (HB(K),K=KK,NK)
  40      CONTINUE
C
C  GO TO NEXT DELAY
C
          WRITE(*,3000)J+1
 3000     FORMAT(2X,'DELAY ',I3,' COMPLETE')
  50 CONTINUE
C
C  RESORT THE BINARY FILE
C
      CALL RESORT(NDELAY,NTIMES)
      RETURN
      END
```

Table 29. RESORT Fortran code.

```
      SUBROUTINE RESORT(NDELAY,NTIMES)
C
C********************* VERSION 1.5 *** 08 JAN 1990 ****************************
C
C  THIS ROUTINE RESORTS FILES GENERATED UNDER FROZEN-IN MODEL
C  FROM ALL TIME SAMPLES FOR EACH DELAY TO ALL DELAY SAMPLES FOR EACH TIME
C
C
C**********************  SET ARRAY SIZES  *************************************
C
      INCLUDE 'CIRF.SIZ'
C
C******************************************************************************
C
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/IMPULS/HA(MTIMES,MDELAY),HB(MTIMES),HC(MDELAY)
      COMPLEX HA,HB,HC
      CHARACTER*80 IDENT
      DIMENSION RDUM1(NDATA1),RDUM2(NDATA2)
      LOGICAL RESET
C
C******  RESET WRITER ROUTINE
C
      RESET = .TRUE.
      CALL WRITER(RESET,NDELAY,NTIMES,IUNIT,HC)
C
C******  READ IMPULSE RESPONSE FUNCTION FROM FILE
C
      REWIND IOFILE
      READ (IOFILE) MDENT,IDENT(1:MDENT)
      READ (IOFILE) NDUM1,(RDUM1(N),N=1,NDUM1)
      READ (IOFILE) NDUM2,(RDUM2(N),N=1,NDUM2)
      DO 20 N=1,NDELAY
          DO 10 I=1,NTIMES,MAXBUF/2
              IP = MIN0(NTIMES,I+MAXBUF/2-1)
              READ(IOFILE) (HA(K,N),K=I,IP)
   10     CONTINUE
   20 CONTINUE
C
C******  WRITE IMPULSE RESPONSE FUNCTION BACK INTO FILE
C
      REWIND IOFILE
      READ (IOFILE) MDENT,IDENT(1:MDENT)
      READ (IOFILE) NDUM1,(RDUM1(N),N=1,NDUM1)
      READ (IOFILE) NDUM2,(RDUM2(N),N=1,NDUM2)
      DO 40 K=1,NTIMES
          DO 30 J=1,NDELAY
              HC(J) = HA(K,J)
   30     CONTINUE
          CALL WRITER(RESET,NDELAY,NTIMES,IOFILE,HC)
   40 CONTINUE
      RETURN
      END
```

Table 30. TAUPSD Fortran code.

```
      FUNCTION TAUPSD(TAU)
C
C********************** VERSION 1.1 *** 16 MAY 1988 ***************************
C
C  THIS FUNCTION CALCULATES THE POWER SPECTRAL DENSITY FOR
C    ISOTROPIC SCATTERING
C    NO EXPLICIT ANTENNA
C
C******************************************************************************
C
      COMMON/REALZN/NDELAY,NTIMES,ALPHA,NO
      COMMON/PSDCOM/OMEGAT,SO
      ARG = (1.0 + ALPHA**2*(OMEGAT - TAU))/(SQRT(2.0)*ALPHA)
      TAUPSD = SO*EXP(0.5/ALPHA**2-TAU)*WITWER(ARG)
      RETURN
      END
```

Table 31. WITWER Fortran code.

```
      FUNCTION WITWER(Z)
C
C********************** VERSION 1.0 *** 25 NOV 1987 ***************************
C
C  WITTWER'S G FUNCTION (DNA 5662D P34)
C
C******************************************************************************
C
      PARAMETER (PI=3.141592654)
      IF(Z .GT. 1.0)THEN
          F = 1.0 - 0.191/Z**2 + 0.197/Z**4 - 0.112/Z**6
          WITWER = SQRT(PI/(2.0*Z))*EXP(-Z**2)*F
          RETURN
      END IF
      IF(Z .LE. 1.0 .AND. Z .GE. -1.0)THEN
          F = -0.675*Z - 0.729*Z**2 - 0.109*Z**3 + 0.031*Z**4
          WITWER = 1.813*EXP(F)
          RETURN
      END IF
      IF(Z .LT. -1.0)THEN
          F = 1.0 + 0.290/Z**2 - 0.178/Z**4 + 0.0014/Z**6
          WITWER = SQRT(PI/ABS(Z))*F
          RETURN
      END IF
      END
```

Table 32. CHANL3 Fortran code.

```
      SUBROUTINE CHANL3
C
C********************** VERSION 2.3 *** 08 JAN 1990 ****************************
C
C  THIS SUBROUTINE CALCULATES IMPULSE RESPONSE FUNCTION USING THE
C  TURBULENT APPROXIMATION WITHOUT ANTENNA EFFECTS.
C
C***********************  SET ARRAY SIZES  **************************************
C
      INCLUDE 'CIRF.SIZ'
C
C*******************************************************************************
C
      COMMON/CHANEL/TAU0,F0
      COMMON/GRDSIZ/DELTAU,TAUMIN,DELTAT,DELTAF
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/POWERS/G(MDELAY),GRDPOW(2)
      COMMON/RANSED/ISEED,JSEED
      COMMON/REALZN/NDELAY,NTIMES,ALPHA,N0
      COMPLEX AWGN,HA(MDELAY),HB(MDELAY),HC(MDELAY)
      PARAMETER (SMALL=1.0E-10)
      LOGICAL RESET
C
C****** RESET WRITER ROUTINE
C
      RESET = .TRUE.
      CALL WRITER(RESET,NDELAY,NTIMES,IOFILE,HC)
C
C******  INITIALIZE 1/F**4 RANDOM NUMBERS
C
      X   = 2.146193/N0
      RHO = EXP(-X)
      AMP = SQRT(TANH(X))
      C1  = RHO
      C2  = SQRT(1.0-RHO**2)
      C3  = C2*AMP
      DO 30 J=1,NDELAY
          HA(J) = AWGN(ISEED,JSEED)
          HB(J) = AMP*AWGN(ISEED,JSEED)
C
C  WARM UP THE 1/F**4 FILTERS
C
          DO 20 NN=1,N0
              HA(J) = C1*HA(J) + C3*HB(J)
              HB(J) = C1*HB(J) + C2*AWGN(ISEED,JSEED)
   20     CONTINUE
   30 CONTINUE
C
C******  START LOOP OVER TIME
C
      DO 50 NT=1,NTIMES
          DO 40 J=1,NDELAY
C
C  SET IMPULSE RESPONSE FUNCTION TO ZERO IF ENERGY IN DELAY BIN IS SMALL
C
```

Table 32. CHANL3 Fortran code (Continued).

```
             IF(G(J) .LT. SMALL*G(1))THEN
                 HC(J) = (0.0,0.0)
                 GO TO 40
             END IF
C
C  UPDATE RANDOM NUMBERS AND GENERATE IMPULSE RESPONSE FUNCTION
C
             HA(J) = C1*HA(J) + C3*HB(J)
             HB(J) = C1*HB(J) + C2*AWGN(ISEED,JSEED)
             HC(J) = SQRT(G(J))*HA(J)/DELTAU
   40    CONTINUE
C
C  WRITE IMPULSE RESPONSE FUNCTION INTO BINARY FILE
C
         CALL WRITER(RESET,NDELAY,NTIMES,IOFILE,HC)
C
C  GO TO NEXT TIME
C
         IF(MOD(NT,NTIMES/NO) .EQ. 0)THEN
             WRITE(*,3000)NT
 3000        FORMAT(2X,'TIME ',I5,' COMPLETE')
         END IF
   50 CONTINUE
      RETURN
      END
```

Table 33. CHANL4 Fortran code.

```
      SUBROUTINE CHANL4
C
C********************* VERSION 2.2 *** 08 JAN 1990 ****************************
C
C  FLAT FADING MODEL WITH 1/F**4 DOPPLER SPECTRUM AND WITHOUT ANTENNA EFFECTS
C
C******************************************************************************
C
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/GRDSIZ/DELTAU,TAUMIN,DELTAT,DELTAF
      COMMON/RANSED/ISEED,JSEED
      COMMON/REALZN/NDELAY,NTIMES,ALPHA,NO
      COMPLEX AWGN,HA,HB,HC
      LOGICAL RESET
C
C******  RESET WRITER ROUTINE
C
      RESET = .TRUE.
      CALL WRITER(RESET,NDELAY,NTIMES,IOFILE,HA)
C
C******  INITIALIZE 1/F**4 RANDOM NUMBERS
C
      X    = 2.146193/NO
      RHO  = EXP(-X)
      AMP  = SQRT(TANH(X))
      C1   = RHO
      C2   = SQRT(1.0-RHO**2)
      C3   = C2*AMP
      HA   = AWGN(ISEED,JSEED)
      HB   = AMP*AWGN(ISEED,JSEED)
C
C******  WARM UP THE 1/F**4 FILTERS
C
      DO 30 NN=1,NO
          HA = C1*HA + C3*HB
          HB = C1*HB + C2*AWGN(ISEED,JSEED)
   30 CONTINUE
C
C******  START LOOP OVER TIME
C
      DO 50 NT=1,NTIMES
C
C  UPDATE RANDOM NUMBERS AND GENERATE IMPULSE RESPONSE FUNCTION
C
          HA = C1*HA + C3*HB
          HB = C1*HB + C2*AWGN(ISEED,JSEED)
          HC = HA/DELTAU
C
C  WRITE IMPULSE RESPONSE FUNCTION INTO BINARY FILE
C
          CALL WRITER(RESET,NDELAY,NTIMES,IOFILE,HC)
          IF(MOD(NT,NTIMES/NO) .EQ. 0)THEN
              WRITE(*,3000)NT
 3000         FORMAT(2X,'TIME ',I5,' COMPLETE')
          END IF
   50 CONTINUE
      RETURN
      END
```

Table 34. TAUGD2 Fortran code.

```
      SUBROUTINE TAUGD2(IFADE,NDRQD)
C
C******************** VERSION 2.1 *** 08 JAN 1990 ****************************
C
C  THIS SUBROUTINE COMPUTES THE ENERGY IN DELAY GRID CELLS AND:
C    STOPS CIRF IF THERE ARE TOO FEW DELAY GRID CELLS
C    PRINTS A WARNING IF THERE ARE TOO MANY DELAY GRID CELLS
C
C*********************  SET ARRAY SIZES  ***********************************
C
      INCLUDE 'CIRF.SIZ'
C
C*****************************************************************************
C
      EXTERNAL PIRF2
      COMMON/CHANEL/TAU0,F0
      COMMON/GRDSIZ/DELTAU,TAUMIN,DELTAT,DELTAF
      COMMON/INTGRL/NTASUB
      COMMON/REALZN/NDELAY,NTIMES,ALPHA,NO
      COMMON/POWERS/G(MDELAY),GRDPOW(2)
      PARAMETER (PI=3.141592654)
      PARAMETER (SMALL=1.0E-10,BIG=5.0)
      LOGICAL STOPER
C
C*****************************************************************************
C
      IF(IFADE .EQ. 2)THEN
          ERQD = 0.95
          OMEGAC = 2.0*PI*F0*SQRT(1.0 + 1.0/ALPHA**2)
      END IF
      IF(IFADE .EQ. 3)THEN
          ERQD = 0.975
          OMEGAC = 2.0*PI*F0
      END IF
      GRDPOW(2) = 0.0
      DO 40 J=1,NDELAY
          TAU  = TAUMIN + (J-1)*DELTAU
          TAU1 = OMEGAC*(TAU - DELTAU/2.0)
          TAU2 = OMEGAC*(TAU + DELTAU/2.0)
          IF(TAU1 .LE. SMALL .AND. TAU2 .GE. BIG)THEN
              G(J) = 1.0
              GRDPOW(2) = 1.0
              GO TO 40
          END IF
          IF(IFADE.EQ.2)G(J) = SIMPSN(PIRF2,TAU1,TAU2,NTASUB)
          IF(IFADE.EQ.3)G(J) = EXP(-TAU1) - EXP(-TAU2)
          GRDPOW(2) = GRDPOW(2) + G(J)
          IF(IFADE .EQ. 3)GRDPOW(1) = GRDPOW(2)
   40 CONTINUE
C
C******  TEST DELAY GRID FOR TOO LITTLE ENERGY OR EXCESS DELAYS
C
      STOPER = .FALSE.
      TAUMAX = 0.0
      NDRQD  = 0
      IF(GRDPOW(2) .LT. ERQD)THEN
```

Table 34. TAUGD2 Fortran code (Continued).

```
C
C  NOT ENOUGH ENERGY IN GRID, CALCULATE REQUIRED MAXIMUM DELAY
C
           STOPER = .TRUE.
           IF(IFADE .EQ. 3)THEN
               TAUMAX = -ALOG(1.0-ERQD)
           ELSE
               TAU1 = OMEGAC*(TAUMIN - 0.5*DELTAU)
               TAU2 = OMEGAC*(TAUMIN + NDELAY*DELTAU - 0.5*DELTAU)
               MAXITR = 1 + ALOG(BIG/TAU2)/ALOG(1.1)
               ETAU1 = SIMPSN(PIRF2,TAU1,TAU2,NDELAY*NTASUB)
               TAU1 = TAU2
               DO 60 ITER=1,MAXITR
                   TAU2 = 1.1*TAU1
                   ETAU2 = ETAU1+SIMPSN(PIRF2,TAU1,TAU2,NTASUB)
                   IF(ETAU2 .GE. ERQD)GO TO 65
                   TAU1  = TAU2
                   ETAU1 = ETAU2
   60          CONTINUE
   65          TAU2 = TAU1 + (ERQD-ETAU1)*(TAU2-TAU1)/(ETAU2-ETAU1)
               TAUMAX = AMAX1(TAUMAX,TAU2)
           END IF
       ELSE
C
C  ENOUGH ENERGY, SEE IF THERE ARE TOO MANY DELAY BINS
C
               ETAU2 = 0.0
               DO 70 J=1,NDELAY
                   ETAU2 = ETAU2 + G(J)
                   IF(ETAU2 .GE. ERQD) GO TO 75
   70          CONTINUE
   75          NDRQD = MAX0(NDRQD,J)
           END IF
   80 CONTINUE
C
C****** NOT ENOUGH ENERGY IN DELAY GRID - STOP CIRF
C
       IF(STOPER)THEN
           TAUINP  = NDELAY*DELTAU
           TAURQD  = TAUMAX/OMEGAC - (TAUMIN - 0.5*DELTAU)
           NDRQD = 1.0 + TAURQD/DELTAU
           WRITE(*,4011)
           WRITE(*,4012)TAUINP,TAURQD,NDELAY,NDRQD
 4011      FORMAT(2X,'******   DELAY GRID ERROR   ******')
 4012      FORMAT(4X,'INSUFFICIENT DELAY IN DELAY GRID'/4X,
     1       'INPUT    DELAY  = ',1PE12.4,' = NDELAY*DELTAU'/4X,
     2       'REQUIRED DELAY  = ',E12.4/4X,
     3       'INPUT    NDELAY = ',I12/4X,
     4       'REQUIRED NDELAY = ',I12,' FOR FIXED DELTAU')
           STOP '  CIRF EXECUTION TERMINATED'
       END IF
C
C****** TOO MANY DELAY BINS - PRINT WARNING
C
       IF(NDRQD .LT. NDELAY)WRITE(*,4015)NDRQD
 4015 FORMAT(/2X,'**********   WARNING   **********'/2X,
     1  'ONLY ',I4,' DELAY BINS ARE REQUIRED'/2X,
     2  '**********************************')
       RETURN
       END
```

Table 35. MEASR2 Fortran code.

```
      SUBROUTINE MEASR2
C
C******************** VERSION 2.2  *** 25 APR 1991 **************************
C
C  THIS SUBROUTINE MEASURES AND WRITES THE STATISTICS
C  OF THE IMPULSE RESPONSE FUNCTION REALIZATIONS GENERATED BY CIRF
C
C*********************  SET ARRAY SIZES  ************************************
C
      INCLUDE 'CIRF.SIZ'
C
C****************************************************************************
C
      CHARACTER*80 IDENT
      LOGICAL RESET
      COMMON/CHANEL/TAU0,F0
      COMMON/IMPULS/HA(MTIMES,MDELAY),HB(MTIMES),HC(MDELAY)
      COMMON/INOUT /IREAD,IWRITE,IOFILE
      COMMON/POWERS/G(MDELAY),GRDPOW(2)
      DIMENSION AMPAVE(MDELAY+1,4),S4(MDELAY+1),CHIBAR(MDELAY+1,2)
      DIMENSION HEADER(NDATA1)
      COMPLEX HA,HB,HC,HD
      PARAMETER (PI=3.141592654,TWOPI=2.0*PI,EULER=0.5772157)
      PARAMETER (X1NORM=-EULER/2.0,X2NORM=PI**2/24.0+EULER**2/4.0)
C
C******  READ REALIZATIONS AND COLLECT STATISTICS
C
      WRITE(IWRITE,2000)
 2000 FORMAT(1H1,1X,'MEASURED PARAMETERS OF REALIZATION')
C
C  READ HEADER RECORDS
C
      REWIND IOFILE
      READ(IOFILE) MDENT,IDENT(1:MIN0(MDENT,80))
      READ(IOFILE) MDATA1,(HEADER(II),II=1,MIN0(MDATA1,NDATA1))
      NTIMES = HEADER(14) + 0.01
      DELTAT = HEADER(15)
      N0     = HEADER(16) + 0.01
      NDELAY = HEADER(20) + 0.01
      JT     = NDELAY + 1
      TAUMIN = HEADER(21)
      DELTAU = HEADER(22)
      RESET  = .TRUE.
      CALL READER(RESET,NDELAY,NTIMES,IOFILE,HC)
C
C  INITIALIZE MEASUREMENTS
C
      DO 20 J=1,JT
          CHIBAR(J,1) = 0.0
          CHIBAR(J,2) = 0.0
          DO 10 M=1,4
              AMPAVE(J,M) = 0.0
   10     CONTINUE
   20 CONTINUE
      AVENRG = 0.0
      AVETAU = 0.0
      SIGTAU = 0.0
      DO 40 KK=1,NTIMES
```

Table 36. MEASR2 Fortran code (Continued).

```
C
C  READ IMPULSE RESPONSE FUNCTION
C
         CALL READER(RESET,NDELAY,NTIMES,IOFILE,HC)
         HB(KK) = (0.0,0.0)
         HD = (0.0,0.0)
C
C  COLLECT STATISTICS VERSUS DELAY
C
         DO 30 J=1,NDELAY
              AMP = CABS(HC(J))*DELTAU
              IF(AMP .GT. 0.0)THEN
                   POW = AMP**2
                   HD = HD + HC(J)*DELTAU
                   HB(KK) = HB(KK) + HC(J)*DELTAU
                   AMPAVE(J,1) = AMPAVE(J,1) + AMP
                   AMPAVE(J,2) = AMPAVE(J,2) + POW
                   AMPAVE(J,3) = AMPAVE(J,3) + AMP**3
                   AMPAVE(J,4) = AMPAVE(J,4) + POW**2
                   AMP = ALOG(AMP)
                   CHIBAR(J,1) = CHIBAR(J,1) + AMP
                   CHIBAR(J,2) = CHIBAR(J,2) + AMP**2
                   TAU = TAUMIN + (J-1)*DELTAU
                   AVENRG = AVENRG + POW
                   AVETAU = AVETAU + POW*TAU
                   SIGTAU = SIGTAU + POW*TAU**2
              END IF
   30    CONTINUE
         AMP = CABS(HD)
         AMPAVE(JT,1) = AMPAVE(JT,1) + AMP
         AMPAVE(JT,2) = AMPAVE(JT,2) + AMP**2
         AMPAVE(JT,3) = AMPAVE(JT,3) + AMP**3
         AMPAVE(JT,4) = AMPAVE(JT,4) + AMP**4
         AMP = ALOG(AMP)
         CHIBAR(JT,1) = CHIBAR(JT,1) + AMP
         CHIBAR(JT,2) = CHIBAR(JT,2) + AMP**2
   40 CONTINUE
C
C******  MEASURED DECORRELATION TIME
C
      CALL RHO(HB,NTIMES,XTAU0)
      TOMEAS = 1.0E30
      IF(XTAU0 .GT. 0.0)TOMEAS = XTAU0*DELTAT
C
C******  MEASURED MOMENTS OF AMPLITUDE VERSUS DELAY
C
      DO 60 J=1,JT
         DO 50 M=1,4
              AMPAVE(J,M) = AMPAVE(J,M)/NTIMES
   50    CONTINUE
         CHIBAR(J,1) = CHIBAR(J,1)/NTIMES
         CHIBAR(J,2) = CHIBAR(J,2)/NTIMES
   60 CONTINUE
      DO 70 J=1,NDELAY
         PJ = G(J)
```

Table 36. MEASR2 Fortran code (Continued).

```
        IF(AMPAVE(J,2)**2 .GT. 0.0 .AND. PJ**2 .GT. 0.0)THEN
            S4(J) = (AMPAVE(J,4)-AMPAVE(J,2)**2)/AMPAVE(J,2)**2
            IF(S4(J) .GT. 0.0)THEN
                S4(J) = SQRT(S4(J))
            ELSE
                S4(J) = 0.0
            END IF
            AMPAVE(J,1) = AMPAVE(J,1)/(0.5*SQRT(PI))/SQRT(PJ)
            AMPAVE(J,2) = AMPAVE(J,2)/PJ
            AMPAVE(J,3) = AMPAVE(J,3)/(0.75*SQRT(PI))/SQRT(PJ)**3
            AMPAVE(J,4) = AMPAVE(J,4)/2.0/PJ**2
            AMP =0.5*ALOG(PJ)
            CHIBAR(J,2) = (CHIBAR(J,2)-2.0*CHIBAR(J,1)*AMP+AMP**2)/
     1        X2NORM
            CHIBAR(J,1) = (CHIBAR(J,1)-AMP)/X1NORM
        ELSE
            S4(J) = 0.0
            AMPAVE(J,1) = 0.0
            AMPAVE(J,2) = 0.0
            AMPAVE(J,3) = 0.0
            AMPAVE(J,4) = 0.0
            CHIBAR(J,1) = 0.0
            CHIBAR(J,2) = 0.0
        END IF
   70 CONTINUE
      PT = GRDPOW(1)
      S4(JT) = SQRT((AMPAVE(JT,4)-AMPAVE(JT,2)**2)/AMPAVE(JT,2)**2)
      AMPAVE(JT,1) = AMPAVE(JT,1)/(0.5*SQRT(PI))/SQRT(PT)
      AMPAVE(JT,2) = AMPAVE(JT,2)/PT
      AMPAVE(JT,3) = AMPAVE(JT,3)/(0.75*SQRT(PI))/SQRT(PT)**3
      AMPAVE(JT,4) = AMPAVE(JT,4)/2.0/PT**2
      AMP = 0.5*ALOG(PT)
      CHIBAR(JT,2) = (CHIBAR(JT,2)-2.0*CHIBAR(JT,1)*AMP+AMP**2)/
     1  X2NORM
      CHIBAR(JT,1) = (CHIBAR(JT,1)-AMP)/X1NORM
C
C******  MEASURED FREQUENCY SELECTIVE BANDWIDTH
C
      AVETAU = AVETAU/AVENRG
      SIGTAU = SIGTAU/AVENRG - AVETAU**2
      FOMEAS = 1.0E30
      IF(SIGTAU .GT. 0.0)FOMEAS = 1.0/(TWOPI*SQRT(SIGTAU))
      AVENRG = AVENRG/NTIMES
C
C******  WRITE MEASURED SIGNAL PARAMETERS VERSUS DELAY
C
      WRITE(IWRITE,2100)
 2100 FORMAT(/2X,'MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY',/4X,
     1  'NORMALIZED TO ENSEMBLE VALUES',/4X,
     2  'POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN',/4X,
     3  'J = T: STATISTICS OF COMPOSITE SIGNAL',//4X,
     4  'J',5X,'POW(J)',5X,'<A>',3X,'<A^2>',3X,'<A^3>',3X,
     5  '<A^4>',6X,'S4',3X,'<CHI>',1X,'<CHI^2>')
```

Table 36. MEASR2 Fortran code (Continued).

```
      DO 80 J=1,NDELAY
          JJ = J - 1
          WRITE(IWRITE,2110)JJ,G(J),(AMPAVE(J,M),M=1,4),S4(J),
     1      CHIBAR(J,1),CHIBAR(J,2)
 2110     FORMAT(2X,I3,1X,1PE10.3,7(1X,0PF7.4))
   80 CONTINUE
      WRITE(IWRITE,2111)GRDPOW(2),(AMPAVE(JT,M),M=1,4),S4(JT),
     1  CHIBAR(JT,1),CHIBAR(JT,2)
 2111 FORMAT(4X,'T',1X,1PE10.3,7(1X,0PF7.4))
C
C******  MEASURED REALIZATION PARAMETERS
C
      TOTLOS = -10.0*ALOG10(AVENRG)
        P0 = 1.0
        SLDB = 0.0
      WRITE(IWRITE,2120)P0,AVENRG,SLDB,TOTLOS,F0,F0MEAS
 2120 FORMAT(/2X,'REALIZATION SIGNAL PARAMETERS:',12X,
     1  'ENSEMBLE',6X,'MEASURED'//4X,
     2  'MEAN POWER OF REALIZATION            = ',F10.6,4X,F10.6/4X,
     3  'LOSS (DB) DUE TO MEAN POWER          = ',F10.3,4X,F10.3/4X,
     4  'FREQUENCY SELECTIVE BANDWIDTH (HZ)   = ',1PE10.3,4X,E10.3)
      WRITE(IWRITE,2121)TAU0,T0MEAS,N0,XTAU0
 2121 FORMAT(4X
     1  'DECORRELATION TIME (SEC)             = ',1PE10.3,4X,E10.3/4X,
     2  'NUMBER OF SAMPLES PER DECORR. TIME   = ',I10,4X,0PF10.3)
      GRDLOS = -10.0*ALOG10(GRDPOW(1))
      WRITE(IWRITE,2130)GRDPOW(1),GRDLOS,GRDPOW(2)
 2130 FORMAT(/2X,'MEAN POWER IN GRID',//4X,
     1  'POWER IN DOPPLER GIRD    = ',F10.6/4X,
     2  'POWER LOSS OF GRID (DB)  = ',F10.3/4X,
     3  'POWER IN DELAY GRID      = ',F10.6)
      RETURN
      END
```

Table 36. READER Fortran code - CIRF Version.

```
      SUBROUTINE READER(RESET,NDELAY,NTIMES,IUNIT,H)
C
C********************* VERSION 1.5 *** 08 JAN 1990 ****************************
C*********************         CIRF VERSION        *****************************
C
C  THIS SUBROUTINE READS A COMPLEX ARRAY FROM A FILE ON UNIT=IUNIT
C  ONE DELAY ARRAY IS READ FOR EACH CALL WITH RESET = .FALSE.
C  THE INITIAL HEADER RECORDS ARE READ WITH RESET = .TRUE.
C
C  INPUTS FROM ARGUMENT LIST:
C
C  RESET  = LOGICAL FLAG
C  NDELAY = NUMBER OF DELAY SAMPLES IN IMPULSE RESPONSE FUNCTION
C         = DIMENSION OF H ARRAY
C  NTIMES = NUMBER OF TIME  SAMPLES IN IMPULSE RESPONSE FUNCTION
C  IUNIT  = UNIT OF FILE
C
C  OUTPUTS TO ARGUMENT LIST:
C
C  H      = IMPULSE RESPONSE FUNCTION ARRAY (DIMENSION NDELAY)
C
C  COMMON BLOCK INPUTS:
C
C  RDATA1 = HEADER RECORD
C
C  PARAMETERS:
C
C  MAXBUF = MAXIMUM BUFFER SIZE (RECORD SIZE IN FILE = MAXBUF + 1)
C  NDATA1 = SIZE OF HEADER RECORD 1
C  NDATA2 = SIZE OF HEADER RECORD 2
C
C*******************************************************************************
C
      INCLUDE 'CIRF.SIZ'
      COMMON/HEADER/RDATA1(NDATA1),RDATA2(NDATA2)
      COMPLEX H(NDELAY),BUFFER(MAXBUF/2)
      DIMENSION HEADER(NDATA1),DUMMY(NDATA2)
      LOGICAL RESET
      CHARACTER*80 IDENT
C
C******   IF RESET = .TRUE. READ HEADER RECORDS AT BEGINNING OF FILE
C
      IF(RESET)THEN
          REWIND IUNIT
          READ(IUNIT) MDENT,IDENT(1:MIN0(MDENT,80))
          READ(IUNIT) MDATA1, (HEADER(II),II=1,MIN0(MDATA1,NDATA1))
          READ(IUNIT) MDATA2, (DUMMY (II),II=1,MIN0(MDATA2,NDATA2))
          MDATA1 = MIN0(MDATA1,NDATA1)
          DO 10 NN=1,MDATA1
C
C  FATAL ERROR IF HEADER RECORD DOES NOT AGREE WITH RDATA1
C
              IF(HEADER(NN) .NE. RDATA1(NN))THEN
                  WRITE(*,4000)NN,HEADER(NN),NN,RDATA1(NN)
```

Table 36. READER Fortran code – CIRF Version (Continued).

```
 4000             FORMAT(2X,'FATAL ERROR REREADING HEADER RECORD',/2X,
     1             'WORD ',I2,' OF HEADER RECORD IS        ',1PE12.5/2X,
     2             'WORD ',I2,' OF HEADER RECORD SHOULD BE ',E12.5)
                  STOP ' READER EXECUTION TERMINATED'
              END IF
   10     CONTINUE
          NTBUF = (MAXBUF/2)/NDELAY
          MAXCX = NTBUF*NDELAY
          NUMBER = 0
          RESET = .FALSE.
          RETURN
      END IF
      NUMBER = NUMBER + 1
C
C  COUNT NUMBER OF CALLS TO PROGRAM
C  FATAL ERROR IF ATTEMPT TO READ BEYOND END-OF-FILE
C
      IF(NUMBER .GT. NTIMES)THEN
          WRITE(*,4001)NUMBER,NTIMES
 4001     FORMAT(2X,'ATTEMPT TO READ BEYOND END-OF-FILE',/2X,
     1       'TIME INDEX = ',I10/2X,'MAX  INDEX = ',I10)
          STOP ' READER EXECUTION TERMINATED'
      END IF
C
C  READ DATA INTO A BUFFER
C
      IF(MOD(NUMBER,NTBUF) .EQ. 1)THEN
          READ(IUNIT) MDATA1,(HEADER(NN),NN=1,MIN0(MDATA1,NDATA1))
          READ(IUNIT) NRLWRD,
     1       (BUFFER(KK),KK=1,MIN0(NRLWRD/2,MAXBUF/2))
          MDATA1 = MIN0(MDATA1,NDATA1)
          DO 30 NN=1,MDATA1
C
C  FATAL ERROR IF HEADER RECORD DOES NOT AGREE WITH RDATA1
C
              IF(HEADER(NN) .NE. RDATA1(NN))THEN
                  WRITE(*,4000)NN,HEADER(NN),NN,RDATA1(NN)
                  STOP ' READER EXECUTION TERMINATED'
              END IF
   30     CONTINUE
      END IF
C
C  READ BUFFERED DATA INTO ARRAY
C
      JOFFST = MOD((NUMBER-1)*NDELAY,MAXCX)
      DO 40 JJ=1,NDELAY
          H(JJ) = BUFFER(JOFFST+JJ)
   40 CONTINUE
      RETURN
      END
```

Table 37. WRITER Fortran code - CIRF Version.

```
      SUBROUTINE WRITER(RESET,NDELAY,NTIMES,IUNIT,H)
C
C******************** VERSION 1.3 *** 08 JAN 1990 ***************************
C********************          CIRF VERSION         ***************************
C
C  THIS SUBROUTINE WRITES A COMPLEX ARRAY INTO A FILE ON UNIT=IUNIT
C
C
C  INPUTS:
C
C  RESET  = LOGICAL FLAG - INITIALIZATION IF RESET = .TRUE.
C  H      = COMPLEX*8 ARRAY
C  NDELAY = NUMBER OF DELAY SAMPLES IN IMPULSE RESPONSE FUNCTION
C  NTIMES = NUMBER OF TIME  SAMPLES IN IMPULSE RESPONSE FUNCTION
C  IUNIT  = UNIT NUMBER OF FILE
C
C  COMMON BLOCK INPUTS:
C
C  RDATA1 = HEADER RECORD
C
C  PARAMETERS:
C
C  MAXBUF = MAXIMUM BUFFER SIZE (RECORD SIZE IN FILE = MAXBUF + 1)
C  NDATA1 = SIZE OF HEADER RECORD 1
C  NDATA2 = SIZE OF HEADER RECORD 2
C
C****************************************************************************
C
      SAVE
      INCLUDE 'CIRF.SIZ'
      LOGICAL RESET
      COMMON/HEADER/RDATA1(NDATA1),RDATA2(NDATA2)
      COMPLEX H(NDELAY),BUFFER(MAXBUF/2)
C
C******  IF RESET .EQ. TRUE THEN INITIALIZE
C
      IF(RESET)THEN
          NTBUF  = (MAXBUF/2)/NDELAY
          NCXWRD = NTBUF*NDELAY
          NRLWRD = 2*NCXWRD
          NUMBER = 0
          RESET = .FALSE.
          RETURN
      END IF
C
C  COUNT NUMBER OF CALLS - ONE PER TIME SAMPLE
C
      NUMBER = NUMBER + 1
      JOFFST = MOD((NUMBER-1)*NDELAY,NCXWRD)
C
C  WRITE DELAY SAMPLES INTO BUFFER
C
      DO 20 JJ=1,NDELAY
          BUFFER(JOFFST+JJ) = H(JJ)
   20 CONTINUE
      IWRITER = MOD(NUMBER,NTBUF)
```

Table 37. WRITER Fortran code - CIRF Version (Continued).

```
C
C  WRITE BUFFER SAMPLES INTO FILE WHEN BUFFER IS FULL
C
      IF(IRITER .EQ. 0)THEN
          WRITE(IUNIT) NDATA1, (RDATA1(NN),NN=1,NDATA1)
          WRITE(IUNIT) NRLWRD, (BUFFER(KK),KK=1,NCXWRD)
      END IF
C
C  WRITE BUFFER SAMPLES INTO FILE WHEN AT END OF REALIZATION
C
      IF(NUMBER .EQ. NTIMES .AND. IRITER .NE. 0)THEN
          LFTCX = MOD(NTIMES,NTBUF)*NDELAY
          LFTRL = 2*LFTCX
          WRITE(IUNIT) NDATA1, (RDATA1(NN),NN=1,NDATA1)
          WRITE(IUNIT) LFTRL, (BUFFER(KK),KK=1,LFTCX)
      END IF
      RETURN
      END
```

Table 38. PIRF2 Fortran code.

```
      FUNCTION PIRF2(TAU)
C
C******************** VERSION 1.1 *** 13 MAY 1988 ****************************
C
C  POWER IMPULSE RESPONSE FUNCTION WITHOUT ANTENNA EFFECTS
C  ISOTROPIC SCATTERING AND FINITE ALPHA
C
C****************************************************************************
C
      COMMON/REALZN/NDELAY,NTIMES,ALPHA,NO
      ARG = (1.0-ALPHA**2*TAU)/(SQRT(2.0)*ALPHA)
      PIRF2 = 0.5*EXP(0.5/ALPHA**2-TAU)*ERFC(ARG)
      RETURN
      END
```

Table 39. FFT Fortran code.

```
      SUBROUTINE FFT(A,M,N,IFFT)
C
C******************** VERSION 1.1 *** 30 JAN 1989 ***************************
C
C     ROUTINE TO DO AN IN-PLACE FAST FOURIER TRANSFORM
C     REFERENCE:  A. V. OPPENHEIM AND R. W. SCHAFER
C                 DIGITAL SIGNAL PROCESSING, PAGE 331
C                 PRENTICE-HALL, 1975
C
C     INPUTS: A    = COMPLEX ARRAY OF VALUES TO BE TRANSFORMED
C             N    = NUMBER OF POINTS = 2**M
C             M    = LOG (BASE 2) OF N
C             IFFT =  1, DO AN FFT
C                  = -1, DO AN INVERSE FFT
C                  = -2, DO AN INVERSE FFT AND NORMALIZE BY 1/NPOINTS
C
C****************************************************************************
C
      PARAMETER (PI=3.141592654)
      COMPLEX A(N),U,W,T
      SGN = 1.0
      IF (IFFT .LT. 0) SGN = -1.0
      NV2 = N/2
      NM1 = N-1
      J = 1
      DO 40 I=1,NM1
          IF (I .GE. J) GO TO 10
          T = A(J)
          A(J) = A(I)
          A(I) = T
   10     K = NV2
   20     IF (K .GE. J) GO TO 30
          J = J - K
          K = K/2
          GO TO 20
   30     J = J + K
   40 CONTINUE
      DO 70 L=1,M
          LE  = 2**L
          LE1 = LE/2
          U = (1.0,0.0)
          W = CMPLX(COS(PI/LE1),-SIN(PI*SGN/LE1))
          DO 60 J=1,LE1
              DO 50 I=J,N,LE
                  IP = I + LE1
                  T = A(IP)*U
                  A(IP) = A(I) - T
                  A(I)  = A(I) + T
   50         CONTINUE
              U = U*W
   60     CONTINUE
   70 CONTINUE
      IF (IFFT .NE. -2) RETURN
      DO 80 I=1,N
          A(I) = A(I)/FLOAT(N)
   80 CONTINUE
      RETURN
      END
```

Table 40. PREPRC Fortran code.

```
      SUBROUTINE PREPRC(IREAD,INPUT)
C
C********************* VERSION 2.0 *** 21 JAN 1988 ***************************
C
C  THIS SUBROUTINE PREPROCESSES THE INPUT FILE AND STRIPS OUT COMMENT LINES
C  COMMENT LINES MUST BEGIN WITH A SEMICOLON (;) IN COLUMN 1
C
C*****************************************************************************
C
      CHARACTER*132 LINE
      OPEN(UNIT=INPUT,STATUS='SCRATCH')
   10 READ(IREAD,1000,END=100)LINE
 1000 FORMAT(A)
      K = INDEX(LINE,';')
      IF(K .EQ. 0)THEN
          NC = LENGTH(LINE)
          WRITE(INPUT,1000)LINE(1:NC)
      END IF
      GO TO 10
  100 REWIND INPUT
      RETURN
      END
```

Table 41. LENGTH Fortran code.

```
      INTEGER FUNCTION LENGTH (STRING)
C
C********************* VERSION 2.0 *** 21 JAN 1988 ***************************
C
C  FUNCTION TO COMPUTE THE LENGTH OF THE NON-BLANK PORTION OF
C  A CHARACTER STRING
C
C*****************************************************************************
C
      CHARACTER  STRING*(*)
      DO 10 II = LEN(STRING),1,-1
          LENGTH = II
          IF (STRING(II:II) .NE. ' ') RETURN
   10 CONTINUE
      LENGTH = 0
      RETURN
      END
```

Table 42. RHO Fortran code.

```
      SUBROUTINE RHO(V,NS,XL)
C
C********************** VERSION 1.2 *** 19 DEC 1988 ****************************
C
C  MEASURE 1/E POINT OF AUTOCORRELATION OF VOLTAGE V
C
C*******************************************************************************
C
      COMPLEX V(NS),CORR
      PARAMETER (EINV = 0.367879441)
      RHO1 = 1.0
      RHO2 = 1.0
      DO 20 M=0,NS-1
          CORR = (0.0,0.0)
          DO 10 N=1,NS-M
              CORR = CORR + V(N)*CONJG(V(N+M))
   10     CONTINUE
          IF(M .EQ. 0)POWER = CABS(CORR)/FLOAT(NS)
          RHO2 = CABS(CORR)/(FLOAT(NS-M)*POWER)
          IF(RHO2 .LT. EINV)THEN
              XL = M - (EINV-RHO2)/(RHO1-RHO2)
              RETURN
          END IF
          RHO1 = RHO2
   20 CONTINUE
      XL = -9999.9
      RETURN
      END
```

Table 43. AWGN Fortran code.

```
      COMPLEX FUNCTION AWGN(ISEED,JSEED)
C
C********************** VERSION 1.1 *** 11 OCT 1988 ****************************
C
C  COMPLEX AWGN WITH ZERO MEAN AND UNITY MEAN POWER
C
C*******************************************************************************
C
      PARAMETER (PI=3.141592654,TWOPI=2.0*PI)
      PHI  = TWOPI*RANDOM(ISEED,JSEED)
      AMP  = SQRT(-ALOG(1.0-RANDOM(ISEED,JSEED)))
      AWGN = CMPLX(AMP*COS(PHI),AMP*SIN(PHI))
      RETURN
      END
```

Table 44. ERFC Fortran code.

```
      FUNCTION ERFC(X)
C
C******************** VERSION 2.2 *** 19 DEC 1988 ***************************
C
C  COMPLEMENTARY ERROR FUNCTION (ABRAMOWITZ AND STEGUN 7.1.26)
C
C*****************************************************************************
C
      PARAMETER (P=0.3275911)
      PARAMETER (A1=0.254829592,A2=-0.284496736,A3=1.421413741,
     1  A4=-1.453152027,A5=1.061405429)
      T1 = ABS(X)
      T  = 1.0/(1.0 + P*T1)
      T1 = EXP(-T1*T1)
      POLY = T*(A1+T*(A2+T*(A3+T*(A4+T*A5))))
      ERFC = T1*POLY
      IF(X .LT. 0.0)ERFC = 2.0 - ERFC
      RETURN
      END
```

Table 45. RANDOM Fortran code.

```
      FUNCTION RANDOM(ISEED,JSEED)
C
C-------------------- Version 1.2 --- 23 Sep 1988 --------------------
C
C     Machine-independent uniformly distributed random number generator
C
C     Algorithm is adapted from the following --
C
C                       I = 69069*I+1, MODULO 2**32
C                       R = I/(2**32)
C
C     The same result is obtained on all machines, regardless of word
C     length, by using two 16-bit seeds and retaining a total of only
C     24 bits in the conversion to the floating-point random number.
C
C       Inputs/outputs
C     ISEED  = high-order part of random number seed (16 bits used)
C     JSEED  = low -order part of random number seed (16 bits used)
C
C       Output
C     RANDOM = uniformly distributed random number (0 to 1)
C
C---------------------------------------------------------------------
C
      JB     = 3533*JSEED + 1
      IA     = 3533*ISEED + JSEED + ISHFT(JB,-16)
      ISEED  = IAND(IA,65535)
      JSEED  = IAND(JB,65535)
      RANDOM = REAL(ISHFT(ISEED,8)+ISHFT(JSEED,-8))/16777216.
      RETURN
      END
```

Table 46. SIMPSN Fortran code.

```
      FUNCTION SIMPSN(FUNCT,A,B,INT)
C
C********************* VERSION 1.0 *** 25 NOV 1987 ****************************
C
C  SIMPSON'S RULE INTEGRATION OF FUNCTION FUNCT FROM A TO B WITH INT POINTS
C
C******************************************************************************
C
      EXTERNAL FUNCT
      IF(INT .EQ. 0)THEN
          SIMPSN = 0.5*(A-B)*(FUNCT(A)+FUNCT(B))
          RETURN
      END IF
      DX = (B-A)/INT
      DD = DX/3.0
      XX = A
      SUM = FUNCT(A) + FUNCT(B)
      IS = 4
      DO 10 I=1,INT-1
          XX = XX + DX
          SUM = SUM + IS*FUNCT(XX)
          IF(IS .EQ. 4)THEN
              IS = 2
          ELSE
              IS = 4
          END IF
   10 CONTINUE
      SIMPSN = DD*SUM
      RETURN
      END
```

D.2 "INCLUDE" FILES.

ACIRF and CIRF "INCLUDE" files, listed in Tables 47 and 48 respectively, are to allow the user to easily change array sizes. The files contain parameter statements that define the maximum allow values for the number of antennas, the number of time samples, the number of delay samples, and the number of K_x and K_y samples. Other parameters define unformatted output file record sizes. INCLUDE file parameters, their functions, and limitations are summarized in Table 49.

In both ACIRF and CIRF, the largest array is dimensioned MDELAY×MTIMES. The user may want to reduce both of these parameters to allow the codes to run efficiently on computers with limited random access memory.

The number of time samples may not be less than 1024, corresponding to $102.4\tau_0$ realizations with 10 samples per τ_0. Thus the minimum value of MTIMES is 1024. If ACIRF is run with multiple antennas with different pointing directions, then to achieve at least 100 decorrelation times in all realizations, it may be necessary to use 2048 time samples. For these cases, the minimum value of MTIMES is 2048.

Table 47. ACIRF "INCLUDE" file.

```
C
C  FOR THE GENERAL MODEL (ACIRF) MKX*MKY MUST BE <= MTIMES*MDELAY
C
      PARAMETER (MDELAY=32,MTIMES=4096)
      PARAMETER (MTENNA=8,MKX=64,MKY=64)
      PARAMETER (MAXBUF=4096,NDATA1=30,NDATA2=32)
```

Table 48. CIRF "INCLUDE" file.

```
      PARAMETER (MDELAY=64,MTIMES=4096)
      PARAMETER (MAXBUF=4096,NDATA1=30,NDATA2=32)
```

Table 49. ACIRF and CIRF "INCLUDE" file parameters.

Parameter	***Function***	***Limitation***
MDELAY	Maximum number of delay samples	$\geq \frac{3.7}{2\pi f_{A,min} \Delta\tau}$
MTIMES	Maximum number of time samples	$\geq$ 1024 $\geq$ 2048 (ACIRF with multiple antennas pointing in different directions, see Eqn. 118)
MTENNA	Maximum number of antennas (ACIRF only)	$\geq$ 8 (there is little advantage in smaller values)
MKY MKX	Maximum number of K_x or K_y samples (ACIRF only)	MKX, MKY $\geq$ 32 MKX$\times$MKY $\leq$ MDELAY$\times$MTIMES (see Eqn. 111 for limitations with multiple antennas)
MAXBUF	Maximum record size in unformatted file	= allowed record size
NDATA1	Size of first header record in unformatted file	$\geq$ 30
NDATA2	Size of second header record in unformatted file	$\geq$ 32

The maximum number of delay samples should be determined by the minimum value of the antenna filtered frequency selective bandwidth and the delay sample size. Equation 126 may be used to estimate the maximum required number of delay samples.

Because ACIRF uses the same array space for both the random angular spectrum of impulse response function and the impulse response function itself, the additional requirement that MKX×MKY must be less than or equal to MDELAY×MTIMES is also imposed. The minimum value for MKX and MKY is 32. A larger value may be necessary to prevent aliasing of the realizations of multiple separated antennas, as determined by Equation 111.

APPENDIX E
FURTHER EXAMPLE INPUT AND FORMATTED OUTPUT FILES FOR ACIRF AND CIRF

Example input and formatted output files for cases not included in Section 5 are presented in this appendix. These examples are for the turbulent model in ACIRF and the AWGN channel model, the turbulent model, and the flat fading model in CIRF.

Table 50. ACIRF turbulent model example input file.

```
;  GENERAL MODEL WITH ANTENNAS - TURBULENT MODEL LIMIT
;
;******  CASE NUMBER
;
;     KASE
      1002/
;
;******  ALPHANUMERIC IDENTIFICATION (80 CHARACTERS OR LESS ENCLOSED IN ' ')
;
'GENERAL MODEL (TURBULENT) WITH ANTENNAS - ACIRF 3.5 USERS GUIDE EXAMPLE'
;
;******  CHANNEL PARAMETERS
;
;   FO(HZ)   TAUO(S)    DLX(M)    DLY(M)       CXT       CYT
     1.0E5    3.0E-3       5.0       5.0       0.0       0.0/
;
;******  ANTENNA PARAMETERS
;
;   NUMANT FREQC(HZ)
         3 2.99793E9/
;
;  BEAMWIDTHS, POSITIONS, ROTATION ANGLES, AND POINTING ANGLES
;  (ONE LINE FOR EACH ANTENNA)
;    ROT = ROTATION ANGLE
;    ELV = ANTENNA BEAM POINTING ANGLE ELEVATION
;    AZM = ANTENNA BEAM POINTING ANGLE AZIMUTH
;
; BWU(DEG)  BWV(DEG)   UPOS(M)   VPOS(M)  ROT(DEG)  ELV(DEG)  AZM(DEG)
    0.5896    0.5896     -10.0       0.0      00.0    0.2948     180.0/
    0.5896    0.5896       0.0       0.0      00.0       0.0       0.0/
    0.5896    0.5896      10.0       0.0      00.0    0.2948       0.0/
;
;******  REALIZATION PARAMETERS
;
;   NDELAY DELTAU(S)    NTIMES       NKX       NKY     ISEED        NO
         8    5.0E-7      1024        32        32   9771975       10/
```

Table 51a. Example ACIRF formatted output for the turbulent model (summary of input and ensemble realization statistics).

```
ACIRF CHANNEL SIMULATION VERSION   3.51
  CASE NUMBER         1002
  TEMPORAL VARIATION FROM GENERAL MODEL
  REALIZATION IDENTIFICATION:
  GENERAL MODEL (TURBULENT) WITH ANTENNAS - ACIRF 3.5 USERS GUIDE EXAMPLE

CHANNEL PARAMETERS
  FREQUENCY SELECTIVE BANDWIDTH  (HZ) =  1.000E+05
  DECORRELATION TIME (SEC)            =  3.000E-03
  X DECORRELATION DISTANCE (M)        =  5.000E+00
  Y DECORRELATION DISTANCE (M)        =  5.000E+00
  TIME-X CORRELATION COEFFICIENT      =  0.000E+00
  TIME-Y CORRELATION COEFFICIENT      =  0.000E+00

REALIZATION PARAMETERS
  NUMBER OF DELAY SAMPLES             =          8
  DELAY SAMPLE SIZE (SEC)             =  5.000E-07
  NUMBER OF TEMPORAL SAMPLES          =       1024
  NUMBER OF DOPPLER FREQUENCY SAMPLES =        162
  NUMBER OF TEMPORAL SAMPLES PER TAU0 =         10
  NUMBER OF KX SAMPLES                =         32
  NUMBER OF KY SAMPLES                =         32
  INITIAL RANDOM NUMBER SEED         =     9771975

ANTENNA PARAMETERS
  NUMBER OF ANTENNAS        =          3
  CARRIER FREQUENCY (HZ)    =  2.998E+09

ANTENNA BEAMWIDTHS, ROTATION ANGLES AND POINTING ANGLES

  N    BWU(DEG)    BWV(DEG)    ROT(DEG)    ELV(DEG)    AZM(DEG)
  1       0.590       0.590       0.000       0.295     180.000
  2       0.590       0.590       0.000       0.000       0.000
  3       0.590       0.590       0.000       0.295       0.000

ANTENNA POSITIONS IN U-V AND X-Y COORDINATES

  N     UPOS(M)     VPOS(M)     XPOS(M)     YPOS(M)
  1 -1.000E+01  0.000E+00 -1.000E+01  0.000E+00
  2  0.000E+00  0.000E+00  0.000E+00  0.000E+00
  3  1.000E+01  0.000E+00  1.000E+01  0.000E+00

ENSEMBLE CHANNEL PARAMETERS AT ANTENNA OUTPUTS

  N    LOSS(DB)       POWER     FA(HZ)   TAUA(SEC)
  1       4.602    0.346611  1.574E+05  3.000E-03
  2       3.141    0.485167  2.061E+05  3.000E-03
  3       4.602    0.346611  1.574E+05  3.000E-03
```

Table 51b. Example ACIRF formatted output for the turbulent model (summary of measured realization statistics for antenna 1).

```
MEASURED PARAMETERS FOR REALIZATION/ANTENNA 1

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
  NORMALIZED TO ENSEMBLE VALUES
  POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
  J = T: STATISTICS OF COMPOSITE SIGNAL

  J    POW(J)      <A>    <A^2>    <A^3>    <A^4>       S4    <CHI> <CHI^2>
  0  1.280E-01  1.0993  1.2165  1.3498  1.4925  1.0085  0.6826  0.9116
  1  8.257E-02  1.0057  0.9514  0.8624  0.7564  0.8194  0.8477  0.7875
  2  5.231E-02  1.0138  0.9776  0.8982  0.7923  0.8112  0.9032  0.9923
  3  3.266E-02  0.9587  0.9281  0.9364  0.9952  1.1449  1.0904  0.8904
  4  2.016E-02  0.9527  0.8667  0.7613  0.6489  0.8531  1.0694  0.9068
  5  1.233E-02  1.0305  1.0328  1.0302  1.0347  0.9695  0.8278  0.8915
  6  7.474E-03  1.0983  1.2134  1.3268  1.4283  0.9696  0.7120  0.9672
  7  4.500E-03  0.9584  0.9411  0.9448  0.9610  1.0817  1.1750  1.0713
  T  3.400E-01  1.0608  1.1216  1.1941  1.2873  1.0231  0.7650  0.8616

REALIZATION SIGNAL PARAMETERS:              ENSEMBLE      MEASURED

  MEAN POWER OF REALIZATION             =   0.346611      0.359224
  TOTAL SCATTERING LOSS (DB)            =      4.602         4.446
  FREQUENCY SELECTIVE BANDWIDTH (HZ)    =  1.574E+05     1.888E+05
  DECORRELATION TIME (SEC)              =  3.000E-03     2.903E-03
  NUMBER OF SAMPLES PER DECORR. TIME    =         10         9.676

MEAN POWER IN GRID

  POWER IN KX-KY GIRD     =    0.339266
  POWER LOSS OF GRID (DB) =       0.093
  POWER IN DELAY GRID     =    0.339998
```

Table 51c. Example ACIRF formatted output for the turbulent model (summary of measured realization statistics for antenna 2).

MEASURED PARAMETERS FOR REALIZATION/ANTENNA 2

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
NORMALIZED TO ENSEMBLE VALUES
POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
J = T: STATISTICS OF COMPOSITE SIGNAL

J	POW(J)	<A>	<A^2>	<A^3>	<A^4>	S4	<CHI>	<CHI^2>
0	2.313E-01	0.9505	0.9359	0.9283	0.9127	1.0411	1.3050	1.3497
1	1.210E-01	0.9334	0.8595	0.7771	0.6859	0.9257	1.2166	1.0518
2	6.334E-02	0.9752	0.9376	0.8996	0.8607	0.9788	1.0185	0.8737
3	3.315E-02	1.0414	1.0615	1.0645	1.0544	0.9336	0.8326	0.9493
4	1.735E-02	0.9121	0.8863	0.9126	0.9803	1.2232	1.4349	1.3369
5	9.078E-03	1.0060	1.0030	1.0040	1.0163	1.0101	0.9349	0.8904
6	4.751E-03	1.1417	1.2363	1.2966	1.3211	0.8536	0.3801	0.5895
7	2.486E-03	1.1123	1.1754	1.1886	1.1606	0.8247	0.5563	0.8461
T	4.824E-01	0.9758	0.9350	0.8958	0.8680	0.9929	1.0217	0.9025

REALIZATION SIGNAL PARAMETERS:		ENSEMBLE	MEASURED
MEAN POWER OF REALIZATION	=	0.485167	0.448320
TOTAL SCATTERING LOSS (DB)	=	3.141	3.484
FREQUENCY SELECTIVE BANDWIDTH (HZ)	=	2.061E+05	2.188E+05
DECORRELATION TIME (SEC)	=	3.000E-03	3.131E-03
NUMBER OF SAMPLES PER DECORR. TIME	=	10	10.438

MEAN POWER IN GRID

POWER IN KX-KY GIRD	=	0.482635
POWER LOSS OF GRID (DB)	=	0.023
POWER IN DELAY GRID	=	0.482437

Table 51d. Example ACIRF formatted output for the turbulent model (summary of measured realization statistics for antenna 3).

```
MEASURED PARAMETERS FOR REALIZATION/ANTENNA 3

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
  NORMALIZED TO ENSEMBLE VALUES
  POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
  J = T: STATISTICS OF COMPOSITE SIGNAL

  J     POW(J)      <A>     <A^2>     <A^3>     <A^4>       S4     <CHI> <CHI^2>
  0  1.280E-01   1.1089   1.2327   1.3620   1.4830   0.9757   0.6545   0.9223
  1  8.257E-02   0.9485   0.9129   0.8868   0.8634   1.0354   1.1949   1.0523
  2  5.231E-02   0.9583   0.9342   0.9130   0.8855   1.0146   1.1954   1.1399
  3  3.266E-02   0.9576   0.9347   0.9489   0.9959   1.1314   1.1209   0.9463
  4  2.016E-02   1.1255   1.2746   1.4073   1.4981   0.9188   0.6631   1.0471
  5  1.233E-02   1.0846   1.1957   1.3215   1.4516   1.0152   0.7534   0.9494
  6  7.474E-03   1.0000   0.9834   0.9651   0.9535   0.9858   0.9430   0.8873
  7  4.500E-03   1.0437   1.0797   1.1262   1.1924   1.0226   0.7982   0.8422
  T  3.400E-01   0.9950   0.9902   0.9762   0.9469   0.9651   1.0231   1.0099

REALIZATION SIGNAL PARAMETERS:               ENSEMBLE       MEASURED

  MEAN POWER OF REALIZATION             =   0.346611       0.365189
  TOTAL SCATTERING LOSS (DB)            =      4.602          4.375
  FREQUENCY SELECTIVE BANDWIDTH (HZ)    =  1.574E+05      1.843E+05
  DECORRELATION TIME (SEC)              =  3.000E-03      3.124E-03
  NUMBER OF SAMPLES PER DECORR. TIME    =         10         10.415

MEAN POWER IN GRID

  POWER IN KX-KY GIRD       =    0.339216
  POWER LOSS OF GRID (DB) =        0.094
  POWER IN DELAY GRID       =    0.339998
```

Table 51e. Example ACIRF formatted output for the turbulent model (ensemble and measured antenna output cross correlation coefficients).

```
ENSEMBLE ANTENNA OUTPUT CROSS CORRELATION

  AMPLITUDE OF CROSS CORRELATION

   N     AMP(N-1)  AMP(N-2)  AMP(N-3)
   1     1.000000  0.131351  0.000298
   2     0.131351  1.000000  0.131351
   3     0.000298  0.131351  1.000000

  PHASE (RADIANS) OF CROSS CORRELATION

   N     PHS(N-1)  PHS(N-2)  PHS(N-3)
   1     0.000000 -0.832184  0.000000
   2    -0.832184  0.000000  0.832184
   3     0.000000  0.832184  0.000000


MEASURED ANTENNA OUTPUT CROSS CORRELATION

  AMPLITUDE OF CROSS CORRELATION

   N     AMP(N-1)  AMP(N-2)  AMP(N-3)
   1     1.000000  0.246541  0.069646
   2     0.246541  1.000000  0.092264
   3     0.069646  0.092264  1.000000

  PHASE (RADIANS) OF CROSS CORRELATION

   N     PHS(N-1)  PHS(N-2)  PHS(N-3)
   1     0.000000 -2.959489  2.643855
   2    -2.959489  0.000000  0.078934
   3     2.643855  0.078934  0.000000

FINAL RANDOM NUMBER SEED =    235708183
```

Table 52. Example CIRF input file for AWGN model.

```
; CONSTANT IMPULSE RESPONSE FUNCTION
;
;******  CASE NUMBER
;
;     KASE
      2000/
;
;******  ALPHANUMERIC IDENTIFICATION (80 CHARACTERS OR LESS ENCLOSED IN ' ')
;
'CONSTANT IMPULSE RESPONSE FUNCTION - ACIRF 3.5 USERS GUIDE EXAMPLE'
;
;******  CHANNEL PARAMETERS
;
;    IFADE    FO(HZ)   TAUO(S)
        0     1.0E30     1.0E30/
;
;******  REALIZATION PARAMETERS
;
;   NDELAY DELTAU(S)    NTIMES     ISEED        NO
       10     5.0E-7      1024   9771975        10/
```

Table 53a. Example CIRF formatted output file for AWGN model (summary of input and ensemble values).

```
CIRF CHANNEL SIMULATION VERSION    1.11
  CASE NUMBER          2000
  CONSTANT IMPULSE RESPONSE FUNCTION
  REALIZATION IDENTIFICATION:
 .CONSTANT IMPULSE RESPONSE FUNCTION - ACIRF 3.5 USERS GUIDE EXAMPLE

CHANNEL PARAMETERS
  FREQUENCY SELECTIVE BANDWIDTH  (HZ) =  1.000E+30
  DECORRELATION TIME (SEC)            =  1.000E+30

REALIZATION PARAMETERS
  NUMBER OF DELAY SAMPLES             =          1
  DELAY SAMPLE SIZE (SEC)             =  5.000E-07
  NUMBER OF TEMPORAL SAMPLES          =       1024
  NUMBER OF TEMPORAL SAMPLES PER TAU0 =         10
  INITIAL RANDOM NUMBER SEED        =      9771975
```

Table 53b. Example CIRF formatted output file for AWGN model (summary of measured realization statistics).

```
MEASURED PARAMETERS OF REALIZATION

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
  NORMALIZED TO ENSEMBLE VALUES
  POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
  J = T: STATISTICS OF COMPOSITE SIGNAL

  J     POW(J)      <A>    <A^2>    <A^3>    <A^4>       S4    <CHI> <CHI^2>
  0  1.000E+00  1.1284  1.0000  0.7523  0.5000  0.0000  0.0000  0.0000
  T  1.000E+00  1.1284  1.0000  0.7523  0.5000  0.0000  0.0000  0.0000

REALIZATION SIGNAL PARAMETERS:               ENSEMBLE       MEASURED

  MEAN POWER OF REALIZATION            =   1.000000       1.000000
  LOSS (DB) DUE TO MEAN POWER          =      0.000          0.000
  FREQUENCY SELECTIVE BANDWIDTH (HZ)   =  1.000E+30      1.000E+30
  DECORRELATION TIME (SEC)             =  1.000E+30      1.000E+30
  NUMBER OF SAMPLES PER DECORR. TIME   =         10      -9999.900

MEAN POWER IN GRID

  POWER IN DOPPLER GIRD    =   1.000000
  POWER LOSS OF GRID (DB)  =      0.000
  POWER IN DELAY GRID      =   1.000000

FINAL RANDOM NUMBER SEED =       9771975
```

Table 54. Example CIRF input file for turbulent model.

```
; FREQUENCY SELECTIVE FADING - TURBULENT MODEL FOR TEMPORAL FLUCTUATIONS
;
;******  CASE NUMBER
;
;     KASE
      2003/
;
;******  ALPHANUMERIC IDENTIFICATION (80 CHARACTERS OR LESS ENCLOSED IN ' ')
;
'TURBULENT MODEL (1/F**4 DOPPLER SPECTRUM) - ACIRF 3.5 USERS GUIDE EXAMPLE'
;
;******  CHANNEL PARAMETERS
;
;    IFADE     F0(HZ)    TAU0(S)
        3       1.0E5     1.0E-2/
;
;******  REALIZATIONS PARAMETERS
;
;   NDELAY DELTAU(S)     NTIMES     ISEED         NO
        12     5.0E-7       1024   9771975        10/
```

Table 55a. Example CIRF formatted output file for turbulent model (summary of input and ensemble values).

```
 CIRF CHANNEL SIMULATION VERSION   1.11
   CASE NUMBER        2003
   TEMPORAL VARIATION FROM TURBULENT MODEL
   REALIZATION IDENTIFICATION:
   TURBULENT MODEL (1/F**4 DOPPLER SPECTRUM) - ACIRF 3.5 USERS GUIDE EXAMPLE

 CHANNEL PARAMETERS
   FREQUENCY SELECTIVE BANDWIDTH  (HZ) =  1.000E+05
   DECORRELATION TIME (SEC)            =  1.000E-02

 REALIZATION PARAMETERS
   NUMBER OF DELAY SAMPLES             =         12
   DELAY SAMPLE SIZE (SEC)             =  5.000E-07
   NUMBER OF TEMPORAL SAMPLES          =       1024
   NUMBER OF TEMPORAL SAMPLES PER TAU0 =         10
   INITIAL RANDOM NUMBER SEED        =      9771975
```

Table 55b. Example CIRF formatted output file for turbulent model (summary of measured realization statistics).

MEASURED PARAMETERS OF REALIZATION

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
NORMALIZED TO ENSEMBLE VALUES
POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
J = T: STATISTICS OF COMPOSITE SIGNAL

J	POW(J)	<A>	<A^2>	<A^3>	<A^4>	S4	<CHI>	<CHI^2>
0	2.696E-01	0.9877	0.9992	1.0278	1.0748	1.0738	1.1067	1.1225
1	1.969E-01	0.9754	0.9509	0.9255	0.8926	0.9872	1.0625	0.9526
2	1.438E-01	1.0386	1.0704	1.1089	1.1563	1.0091	0.8190	0.8479
3	1.051E-01	1.0468	1.1132	1.1853	1.2542	1.0120	0.8976	1.0554
4	7.673E-02	1.0363	1.0904	1.1480	1.1990	1.0084	0.9302	1.0605
5	5.604E-02	0.8994	0.8016	0.6955	0.5862	0.9080	1.3973	1.2260
6	4.093E-02	0.9553	0.9342	0.9197	0.9022	1.0332	1.2135	1.1318
7	2.990E-02	1.0236	1.0220	0.9995	0.9632	0.9189	0.8916	0.9646
8	2.184E-02	1.1129	1.2397	1.3807	1.5301	0.9956	0.6246	0.8783
9	1.595E-02	1.0120	1.0241	1.0563	1.1145	1.0608	0.9110	0.8723
10	1.165E-02	0.9946	0.9360	0.8451	0.7375	0.8268	0.8957	0.8183
11	8.509E-03	1.0026	0.9930	0.9678	0.9285	0.9399	0.9693	0.9444
T	9.769E-01	0.9690	0.9430	0.9181	0.8869	0.9974	1.1070	1.0150

REALIZATION SIGNAL PARAMETERS:		ENSEMBLE	MEASURED
MEAN POWER OF REALIZATION	=	1.000000	0.987661
LOSS (DB) DUE TO MEAN POWER	=	0.000	0.054
FREQUENCY SELECTIVE BANDWIDTH (HZ)	=	1.000E+05	1.245E+05
DECORRELATION TIME (SEC)	=	1.000E-02	1.056E-02
NUMBER OF SAMPLES PER DECORR. TIME	=	10	10.564

MEAN POWER IN GRID

POWER IN DOPPLER GIRD = 0.976946
POWER LOSS OF GRID (DB) = 0.101
POWER IN DELAY GRID = 0.976946

FINAL RANDOM NUMBER SEED = 1368273319

Table 56. Example CIRF input file for flat fading model.

```
;  FLAT RAYLEIGH FADING
;
;******  CASE NUMBER
;
;     KASE
      2004/
;
;******  ALPHANUMERIC IDENTIFICATION (80 CHARACTERS OR LESS ENCLOSED IN ' ')
;
'FLAT FADING (1/F**4 DOPPLER SPECTRUM) - ACIRF 3.5 USERS GUIDE EXAMPLE'
;
;******  CHANNEL PARAMETERS
;
;    IFADE     F0(HZ)    TAU0(S)
         4      1.0E5     1.0E-2/
;
;******  REALIZATIONS PARAMETERS
;
;   NDELAY DELTAU(S)    NTIMES     ISEED        NO
         1     2.5E-7      1024   9771975        10/
```

Table 57a. Example CIRF formatted output file for flat fading model (summary of input and ensemble values).

```
CIRF CHANNEL SIMULATION VERSION   1.11
  CASE NUMBER       2004
  FLAT FADING WITH 1/F**4 DOPPLER SPECTRUM
  REALIZATION IDENTIFICATION:
  FLAT FADING (1/F**4 DOPPLER SPECTRUM) - ACIRF 3.5 USERS GUIDE EXAMPLE

CHANNEL PARAMETERS
  FREQUENCY SELECTIVE BANDWIDTH  (HZ) =  1.000E+30
  DECORRELATION TIME (SEC)            =  1.000E-02

REALIZATION PARAMETERS
  NUMBER OF DELAY SAMPLES             =          1
  DELAY SAMPLE SIZE (SEC)             =  2.500E-07
  NUMPER OF TEMPORAL SAMPLES          =       1024
  NUMBER OF TEMPORAL SAMPLES PER TAU0 =         10
  INITIAL RANDOM NUMBER SEED        =      9771975
```

Table 57b. Example CIRF formatted output file for flat fading model (summary of measured realization statistics).

```
MEASURED PARAMETERS OF REALIZATION

MOMENTS OF VOLTAGE AMPLITUDE VERSUS DELAY
  NORMALIZED TO ENSEMBLE VALUES
  POW(J) = ENSEMBLE POWER IN J-TH DELAY BIN
  J = T: STATISTICS OF COMPOSITE SIGNAL

  J     POW(J)      <A>     <A^2>    <A^3>    <A^4>       S4    <CHI> <CHI^2>
  0  1.000E+00  0.9753   0.9418   0.8988   0.8471   0.9540   1.0720  1.0081
  T  1.000E+00  0.9753   0.9418   0.8988   0.8471   0.9540   1.0720  1.0081

REALIZATION SIGNAL PARAMETERS:              ENSEMBLE       MEASURED

  MEAN POWER OF REALIZATION            =    1.000000       0.941754
  LOSS (DB) DUE TO MEAN POWER          =       0.000          0.261
  FREQUENCY SELECTIVE BANDWIDTH (HZ)   =   1.000E+30      1.056E+09
  DECORRELATION TIME (SEC)             =   1.000E-02      9.724E-03
  NUMBER OF SAMPLES PER DECORR. TIME   =          10          9.724

MEAN POWER IN GRID

  POWER IN DOPPLER GIRD    =   1.000000
  POWER LOSS OF GRID (DB) =      0.000
  POWER IN DELAY GRID      =   1.000000

FINAL RANDOM NUMBER SEED =   -226955153
```

APPENDIX F
LIST OF ACRONYMS

ACIRF	–	Antenna/channel impulse response function
dB	–	Decibel
CIRF	–	Channel impulse response function
DFT	–	Discrete Fourier transform
DNA	–	Defense Nuclear Agency
FFT	–	Fast Fourier transform
FSK	–	Frequency-shift keying
GPSD	–	Generalized power spectral density
MHz	–	Million Hertz (10^6 cycles per second)
MRC	–	Mission Research Corporation
PSD	–	Power spectral density
PSK	–	Phase-shift keying
RCIRF	–	Radar/channel impulse response function
RF	–	Radio frequency

APPENDIX G
LIST OF SYMBOLS

Symbol			Page (where first used)
$A(\rho)$	=	Aperture weighting function	27
a	=	Amplitude of the impulse response function	97
a_j	=	Amplitude of the j^{th} delay bin realization	65
A_x	=	Antenna pointing factor	40
A_y	=	Antenna pointing factor	40
a_0	=	Beamwidth scale factor	21
$\boldsymbol{B}$	=	Geomagnetic field (see Fig. 1)	4
C	=	Magnitude of space-time correlation coefficient	25
C_{pt}	=	p-direction space-time correlation coefficient	30
C_{qt}	=	q-direction space-time correlation coefficient	30
C_{xt}	=	x-direction space-time correlation coefficient	4
C_{yt}	=	y-direction space-time correlation coefficient	4
$\cos(x)$	=	Cosine function	5
D	=	Circular antenna diameter	14
d	=	Differential operator	1
D_ξ	=	Rectangular antenna size in ξ-direction	15
d_x	=	Maximum x-direction separation of antennas	42
d_y	=	Maximum y-direction separation of antennas	42
$\boldsymbol{E}$	=	Complex envelope of received electric field	3
e	=	Base of natural logarithms (2.71828$\cdots$)	4
$E_A(k_x,k_y)$	=	Mean power in an angle grid cell at antenna output	75
$E_A(k_x,k_y,k_D)$	=	Mean power in an angle-Doppler frequency grid cell at antenna output	34
$E_D(k_D)$	=	Mean power in a Doppler frequency grid cell	35
$E_I(k_x,k_y)$	=	Mean incident power in an angle grid cell	77
$E_{KC}(k_x,k_y)$	=	Mean power in an angle grid cell	35
$E_{KD}(k_x,k_y,k_D)$	=	Mean signal power in an angular-Doppler grid cell	34
$E_m(k_x,k_y)$	=	Mean power in a $K_x - K_y$ grid cell for the m^{th} antenna	66
$E_p(k_p)$	=	Mean signal power in a K_p grid cell	35
$E_q(k_q)$	=	Mean signal power in a K_q grid cell	35
$E_1(k_x,k_y)$	=	Mean power of angular grid cell for algorithm 1	77
$E_2(k_x,k_y)$	=	Mean power of angular grid cell for algorithm 2	77

LIST OF SYMBOLS (Continued)

LIST OF SYMBOLS (Continued)

Symbol		Page
$K_{p,start}$	= Starting point of power centroid line	91
$k_{p,start}$	= Starting index of power centroid line	93
$K_{p,stop}$	= Stopping point of power centroid line	91
$k_{p,stop}$	= Stopping index of power centroid line	93
$K_{p,1}$	= Limit of the $K_p - K_q$ region for calculation of E_{KC}	88
$K_{p,2}$	= Limit of the $K_p - K_q$ region for calculation of E_{KC}	88
$K_{p,3}$	= Limit of the $K_p - K_q$ region for calculation of E_{KC}	88
$K_{p,4}$	= Limit of the $K_p - K_q$ region for calculation of E_{KC}	88
K_q	= Component of $\boldsymbol{K}_\perp$ in p–q–z coordinate system	30
k_q	= Index of K_q grid	35
$K_{q,L}$	= Power centroid line K_q value	94
$k_{q,L}$	= Power centroid line K_q index	94
$K_{q,max}$	= Limit of K_q grid	84
$k_{q,max}$	= Maximum K_q index	93
$K_{q,start}$	= Starting point of power centroid line	91
$K_{q,stop}$	= Stopping point of power centroid line	91
$K_{q,1}$	= Limit of the $K_p - K_q$ region for calculation of E_{KC}	88
$K_{q,2}$	= Limit of the $K_p - K_q$ region for calculation of E_{KC}	88
$K_{q,3}$	= Limit of the $K_p - K_q$ region for calculation of E_{KC}	88
$K_{q,4}$	= Limit of the $K_p - K_q$ region for calculation of E_{KC}	88
k_T	= Index of time grid	37
K_u	= Component of $\boldsymbol{K}_\perp$ in u–v–z coordinate system	14
K_v	= Component of $\boldsymbol{K}_\perp$ in u–v–z coordinate system	14
K_x	= Component of $\boldsymbol{K}_\perp$ in x–y–z coordinate system	6
k_x	= Index of K_x grid	34
K_x'	= Dummy K_x variable	33
$K_{x,max}$	= Limit of K_x grid	41
$K_{x,1}$	= Limit of the $K_x - K_y$ region for calculation of E_{KC}	88
$k_{x,1}$	= Lower limit of K_x grid index	87
$K_{x,2}$	= Limit of the $K_x - K_y$ region for calculation of E_{KC}	88
$k_{x,2}$	= Upper limit of K_x grid index	87
K_y	= Component of $\boldsymbol{K}_\perp$ in x–y–z coordinate system	6
k_y	= Index of K_y grid	34
K_y'	= Dummy K_y variable	33
$K_{y,max}$	= Limit of K_y grid	41

LIST OF SYMBOLS (Continued)

Symbol			Page
$K_{y,1}$	=	Limit of the K_x–K_y region for calculation of E_{KC}	88
$k_{y,1}$	=	Lower limit of K_y grid index	87
$K_{y,2}$	=	Limit of the K_x–K_y region for calculation of E_{KC}	88
$k_{y,2}$	=	Upper limit of K_y grid index	87
$\boldsymbol{K}_0$	=	Antenna pointing direction vector	10
K_0	=	Magnitude of antenna pointing direction vector	76
$\boldsymbol{K}_{0,m}$	=	Antenna pointing direction vector for m^{th} antenna	65
K_{0x}	=	x-component of $\boldsymbol{K}_0$ in x–y–z coordinate system	14
K_{0y}	=	y-component of $\boldsymbol{K}_0$ in x–y–z coordinate system	14
$\boldsymbol{K}_\perp$	=	$\boldsymbol{K}$ vector for positions in plane normal to line-of-sight	6
$\boldsymbol{K}_\perp'$	=	Dummy $\boldsymbol{K}_\perp$ vector	28
L	=	Scattering loss (in decibels)	16
ℓ_A	=	Decorrelation distance at antenna output	76
ℓ_{Ax}	=	x-direction decorrelation distance at antenna output	19
ℓ_{Ay}	=	y-direction decorrelation distance at antenna output	19
ℓ_p	=	p-direction decorrelation distance	30
ℓ_q	=	q-direction decorrelation distance	30
ℓ_x	=	Channel x-direction decorrelation distance	4
ℓ_y	=	Channel y-direction decorrelation distance	4
ℓ_0	=	Minimum channel decorrelation distance	5
$\ln(x)$	=	Natural logarithm	14
$\log_{10}(x)$	=	Base 10 logarithm	16
$M(\omega)$	=	Spectrum of transmitted modulation	47
$m(t)$	=	Transmitted modulation	1
$\max(x,y)$	=	Maximum function (equal to the larger of x and y)	42
m_D	=	Dummy Doppler frequency index	85
$\min(x,y)$	=	Minimum function (equal to the smaller of x and y)	93
m_x	=	x-direction Doppler shift index	32
$M_x(k_D)$	=	Cumulative x-direction Doppler shift index	85
m_y	=	y-direction Doppler shift index	32
$M_y(k_D)$	=	Cumulative y-direction Doppler shift index	85
N_D	=	Number of Doppler frequency grid cells	34
$N_{D,min}$	=	Minimum required number of Doppler frequency grid cells	44
N_F	=	Number of frequency samples	48
N_T	=	Number of time samples	37

LIST OF SYMBOLS (Continued)

Symbol		Page
N_x	= Number of K_x grid cells	34
N_y	= Number of K_y grid cells	34
N_τ	= Number of delay samples	37
N_0	= Number of samples per decorrelation time	40
P	= Instantaneous power level of a realization	99
P_A	= Mean power in the GPSD at the output of an antenna	16
P_j	= Mean power in j^{th} delay grid cell	45
$P_{j,m}$	= Mean power in j^{th} delay grid cell for m^{th} antenna	65
P_K	= Mean power in angular grid	76
$P_{K,m}$	= Mean power in angular grid for m^{th} antenna	66
P_0	= Mean power in the GPSD	7
P_τ	= Mean power in delay grid	45
$P_{\tau,m}$	= Mean power in delay grid for m^{th} antenna	66
Q	= Antenna filtering factor for isotropic scattering	21
q	= Axial ratio of striations	5
Q_P	= Pointing term in filtered frequency selective bandwidth	17
Q_{Px}	= K_{0x} term in filtered frequency selective bandwidth	17
Q_{Py}	= K_{0y} term in filtered frequency selective bandwidth	17
Q_{Pxy}	= $K_{0x}K_{0y}$ term in filtered frequency selective bandwidth	17
Q_x	= Antenna filtering factor	16
Q_y	= Antenna filtering factor	16
Q_{xy}	= Antenna filtering factor	16
Q_0	= Antenna filtering factor	16
$r(t)$	= Received signal at time t	1
$r(\tau,t)$	= Received signal at delay τ and time t	48
R_c	= Chip rate	48
$S(\kappa)$	= One-dimensional GPSD versus $K_x \ell_x$, $K_y \ell_y$, or $\tau_0 \omega_D$	39
$S(\mathbf{K}_\perp,\tau,\omega_D)$	= Generalized power spectral density (GPSD)	6
$S_A(\mathbf{K}_\perp,\tau,\omega_D)$	= GPSD of signal at output of an antenna	10
$S_A(K_x)$	= Angular spectrum at antenna output	40
$S_A(\tau)$	= Delay spectrum at antenna output	45
$S_{A,m}(\tau)$	= Delay spectrum at output of m^{th} antenna	65
$S_A(\omega_D)$	= Doppler frequency spectrum at antenna output	43
$S_D(\omega_D)$	= Doppler frequency spectrum of the GPSD	6
$S_K(\mathbf{K}_\perp)$	= Angular spectrum of the GPSD	75

LIST OF SYMBOLS (Continued)

Symbol			Page
$S_{KC}(\boldsymbol{K}_\perp)$	=	Angular spectrum used in channel modeling	32
$S_{KC}(K_p)$	=	One-dimensional angular spectrum used in channel modeling	84
$S_{KD}(\boldsymbol{K}_\perp,\omega_D)$	=	Angle-Doppler frequency spectrum of the GPSD	29
$S_{KS}(\boldsymbol{K}_\perp)$	=	Doppler shifted version of $S_{KC}(\boldsymbol{K}_\perp)$	33
$S_{K\tau}(\boldsymbol{K}_\perp,\tau)$	=	Angle-delay spectrum of the GPSD	6
$S_{\tau D}(\tau,\omega_D)$	=	Delay-Doppler scattering function	10
S_4	=	Scintillation index	65
sign (x)	=	Sign function (equal to the sign of argument)	93
sin (x)	=	Sine function	4
t	=	Relative time	1
T_c	=	Chip duration	47
$T_{dur}(P)$	=	Duration of fades below the power level P	101
T_S	=	Sample period of realization	96
$T_{sep}(P)$	=	Separation of fades below the power level P	101
t_1	=	Time 1	3
t_2	=	Time 2	3
tan (x)	=	Tangent function	30
$\tan^{-1}(x)$	=	Inverse tangent function	83
$\hat{\boldsymbol{u}}$	=	Unit vector in antenna u–v–z coordinate system (see Fig. 5)	13
u_0	=	u-position of antenna in u–v–z coordinate system	57
$\hat{\boldsymbol{v}}$	=	Unit vector in antenna u–v–z coordinate system (see Fig. 5)	13
v_0	=	v-position of antenna in u–v–z coordinate system	57
x	=	Relative x-coordinate in plane normal to line-of-sight	3
$\hat{\boldsymbol{x}}$	=	Unit vector in scattering propagation system (see Fig. 1)	4
x_0	=	x-component of vector ρ_0	32
x_1	=	x-coordinate of position 1 in plane normal to line-of-sight	3
x_2	=	x-coordinate of position 2 in plane normal to line-of-sight	3
y	=	Relative y-coordinate in plane normal to line-of-sight	3
$\hat{\boldsymbol{y}}$	=	Unit vector in propagation coordinate system (see Fig. 1)	5
y_0	=	y-component of vector ρ_0	32
y_1	=	y-coordinate of position 1 in plane normal to line-of-sight	3
y_2	=	y-coordinate of position 2 in plane normal to line-of-sight	3
$\hat{\boldsymbol{z}}$	=	Unit vector pointing along line-of-sight (see Fig. 1)	4

LIST OF SYMBOLS (Continued)

Symbol		Page
α	= Delay parameter in the GPSD	4
α_u	= u-direction antenna beam pattern parameter	14
α_v	= v-direction antenna beam pattern parameter	14
α_ξ	= ξ-direction antenna beam pattern parameter	15
$\Gamma(x,y,\omega,t)$	= Two-position, two-frequency, two-time mutual coherence function	3
γ	= Euler's constant (0.5772157···)	66
$\Gamma_A(t)$	= Temporal coherence function at antenna output	18
$\Gamma_A(x)$	= Spatial coherence function for x-direction at antenna output	18
$\Gamma_A(y)$	= Spatial coherence function for y-direction at antenna output	19
$\delta(x)$	= Dirac delta-function	7
$\delta_{m,n}$	= Kronecker delta symbol	38
ΔK_p	= Angular grid K_p cell size	36
ΔK_q	= Angular grid K_q cell size	36
ΔK_x	= Angular grid K_x cell size	32
ΔK_y	= Angular grid K_y cell size	32
Δt	= Time sample size	37
$\Delta \tau$	= Delay sample size	37
$\Delta \omega$	= Frequency grid sample size	48
$\Delta \omega_D$	= Doppler frequency grid cell size	34
ε_x	= K_x grid shift residual	32
ε_y	= K_y grid shift residual	32
ζ_0	= Fraction of signal power in a grid	39
θ	= Angle measured from line-of-sight	8
ϑ	= Rotation angle between x–y–z and p–q–z coordinate systems	30
θ_x	= Scattering angle about the propagation x-axis	6
θ_y	= Scattering angle about the propagation y-axis	6
Θ_0	= Pointing direction elevation angle (see Fig. 5)	13
θ_0	= Antenna 3-dB beamwidth	14
$\theta_{0\xi}$	= Antenna 3-dB beamwidth about ξ-direction	14
κ	= Normalized variable ($K_x \ell_x$, $K_y \ell_y$, or $\tau_0 \omega_D$)	39
κ_{max}	= Maximum value of κ variable	39
$\kappa_{D,max}$	= Maximum value of κ for Doppler frequency grid	40
$\kappa_{K,max}$	= Maximum value of κ for angular grids	40

LIST OF SYMBOLS (Continued)

Symbol			Page
λ	=	RF wavelength	6
Λ_x	=	Asymmetry factor	4
Λ_y	=	Asymmetry factor	4
μ_k	=	First moment of amplitude for the k^{th} realization	98
ξ	=	Arbitrary direction in plane normal to line-of-sight	14
ξ_N	=	Complex, normally-distributed, zero-mean, unity-power random number	31
ξ_{Ux}	=	Uniformly distributed random number on the interval [0,1)	37
ξ_{Uy}	=	Uniformly distributed random number on the interval [0,1)	37
ξ_{U1}	=	Uniformly distributed random number on the interval [0,1)	38
ξ_{U2}	=	Uniformly distributed random number on the interval [0,1)	38
π	=	Pi (3.141592654$\cdots$)	3
ρ	=	Two-dimensional position vector in the plane normal to line-of-sight	27
ρ_0	=	Antenna position in plane normal to line-of-sight	27
σ_θ	=	Angle-of-arrival jitter standard deviation	8
$\sigma_{\theta x}$	=	Angle-of-arrival jitter standard deviation about the x-axis	8
$\sigma_{\theta y}$	=	Angle-of-arrival jitter standard deviation about the y-axis	8
σ_τ	=	Time-of-arrival jitter standard deviation	7
τ	=	Delay (relative time-of-arrival)	1
τ'	=	Dummy delay	28
τ_A	=	Decorrelation time at antenna output	18
$\tau_{A,max}$	=	Maximum τ_A for all antennas	44
$\tau_{A,min}$	=	Minimum τ_A for all antennas	43
τ_{max}	=	Maximum required value of delay grid	45
τ_{min}	=	Minimum delay for frozen-in model without antennas	70
τ_0	=	Channel decorrelation time	4
Φ	=	Penetration angle (see Fig. 1)	4
Φ_0	=	Pointing direction azimuth angle (see Fig. 5)	13
Ψ	=	Rotation angle (see Fig. 5)	13
χ	=	Log amplitude	65

LIST OF SYMBOLS (Continued)

Symbol		Page
ω	= Radian frequency	1
ω_A	= Mean Doppler shift at antenna output	18
ω_{coh}	= Coherence bandwidth	4
ω_D	= Doppler radian frequency	6
ω_D'	= Dummy Doppler radian frequency	28
$\omega_{D,max}$	= Limit of ω_D grid	43
ω_1	= Radian frequency 1	3
ω_2	= Radian frequency 2	3
∞	= Infinity	2

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